1. **First question**

a. Using dimensional analysis, determine the periodic time of the simple pendulum. (10 Marks)

b. Proof that the work=1/2 x stress x strain (5 Marks)

c. Proof that poisson's ratio=1/2 (5 Marks)

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2. **Second question**

a. Derive an expression for moment of inertia of solid sphere.

(5 Marks)

b. Find the resultant wave of the following two waves:-

y1=Asin(kx-wt)

y2=Asin(kx+wt) (Standing waves)

(5 Marks)

c. Derive the Bernouli equation in case of ideal fluid.

(١٠ Marks)

Kind regards

Solution

1.a Assume that t, m, £ and g are related through the equation:

$$t \propto m^x \ell^y g^z$$

$$t = k\, m^x \ell^y g^z$$

By using the dimensional method

$T = M^x L^y (LT^{-2})^z$

$M^0 L^{\circ} T^1 = M^x L^{y+z} T^{-2z}$

Comparison the powers of M, L and T on both sides

x = 0, y + z-0, -2z = l

Solving the three equations,

$$x = 0, \quad y = \tfrac{1}{2}, \quad z = -\tfrac{1}{2}$$

$$\therefore\ t = k\sqrt{\frac{\ell}{g}}$$

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1.b When a body is deformed by the application of external forces, the body gets strained. The work done is stored in the body in the form of energy and is called the energy of strain. Consider a wire of length ℓ, area cross section A and Young's modulus of elasticity Y, see Fig. (6).

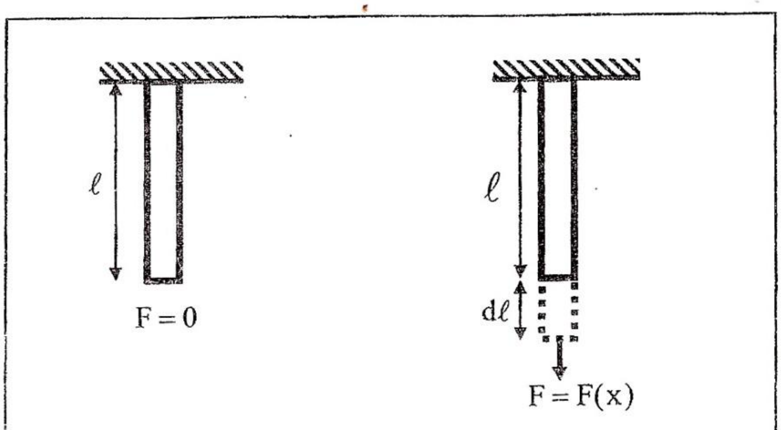


Figure (6)

Let $d\ell$ be the increase in *length* when a stretching force F is applied. Therefore, work done W

$$W = \int dW = \int_0^{d\ell} F(x)\, dx$$

But $F = kx$ so

$$W = \int_0^{d\ell} kx\, dx = \frac{1}{2}kx^2\Big|_0^{d\ell}$$

$$= \frac{1}{2}k(d\ell)^2 = \frac{1}{2}F(d\ell)$$

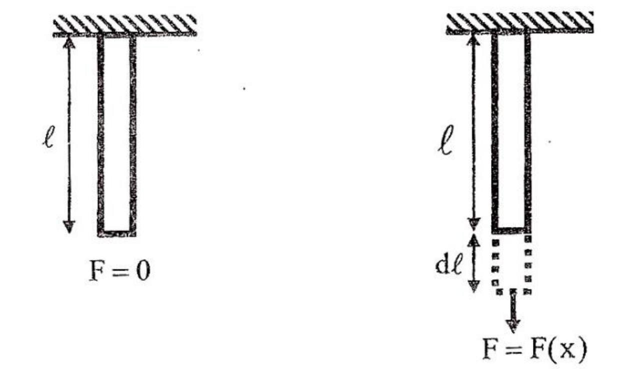
work done per unit volume,

$$w = \frac{W}{V} = \frac{W}{A\ell}$$

$$w = \frac{F(d\ell)}{2A\ell} = \frac{1}{2} \times \frac{F}{A} \times \frac{d\ell}{\ell}$$

$$w = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

1.c When a body is deformed by the application of external forces, the body gets strained. The work done is stored in the body in the form of energy and is called the energy of strain. Consider a wire of length ℓ, area cross section A and Young's modulus of elasticity Y, see Fig. (6).



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$$w = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

The initial volume of the wire is

$$V = \pi r^2 \ell$$

If the volume of the wire remains unchanged (dV = 0) after the force has been applied, then

$$dV = 0$$

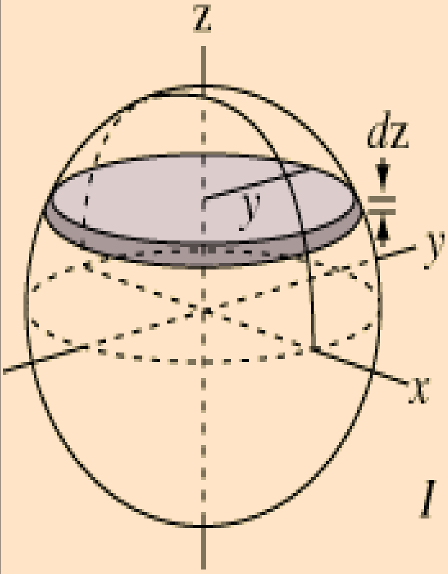
$$0 = \pi(r^2\, d\ell + 2r\, dr\, \ell)$$

$$r\, d\ell = -2\ell\, dr$$

$$\therefore\ \sigma = \frac{-dr/r}{d\ell/\ell} = \frac{1}{2}$$

2. a

The expression for the moment of inertia of a sphere can be developed by summing the moments of infintesmally thin disks about the z axis. The moment of inertia of a thin disk is



$$dI = \frac{1}{2}y^2 dm = \frac{1}{2}y^2 \rho dV = \frac{1}{2}y^2 \rho\pi y^2 dz$$

and the integral becomes

$$I = \frac{1}{2}\rho\pi \int_{-R}^{R} y^4 dz = \frac{1}{2}\rho\pi \int_{-R}^{R} (R^2 - z^2)^2 dz = \frac{8}{15}\rho\pi R^5$$

Radius = R

Mass = M

Density $= \rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$

Substituting the density expression gives

$$I = \frac{8}{15}\left[\frac{M}{\frac{4}{3}\pi R^3}\right]\pi R^5 = \frac{2}{5}MR^2$$

2. b Let us write the equations of two waves propagating along the x-axis in opposite directions:

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

Adding these two equations

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

From the trigonometric relations

$$\sin a + \sin b = 2\sin\tfrac{1}{2}(a+b)\cos\tfrac{1}{2}(a-b)$$

We obtain

$$y = 2A\sin\tfrac{1}{2}(2kx)\cos\tfrac{1}{2}(-2\omega t)$$

$$y = 2A\sin kx\cos(\omega t)$$

This is the equation of standing waves. The amplitude depending on x as

$$2A\sin(kx)$$

At the points whose coordinates comply with the condition

$$kx = (n + \tfrac{1}{2})\pi\ , \qquad (n = 0,\ 1,\ 2,\ \ldots)$$

But $k = \dfrac{2\pi}{\lambda}$, so

$$x_{anti} = (n + \tfrac{1}{2})\frac{\lambda}{2}$$

The amplitude of the oscillations reaches its maximum value. These points are known as antinodes of the standing wave , fig (4).

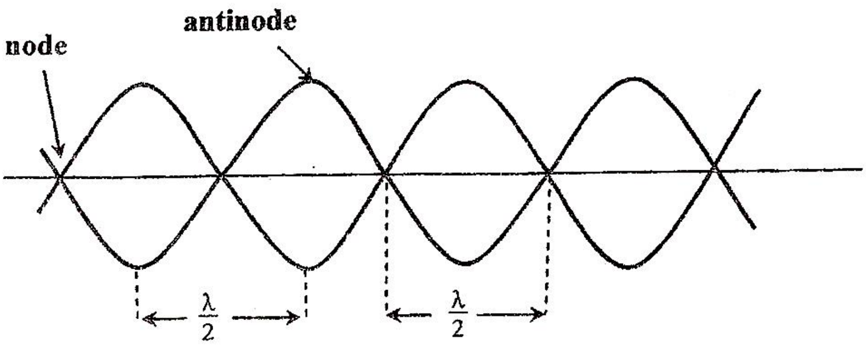
At the points whose coordinates comply with the condition.

$$kx = n\pi \quad , \qquad (n = 0,\ 1,\ 2,\ \ldots)$$

or

$$x_{node} = n\frac{\pi}{k} = n\frac{\lambda}{2}$$

The amplitude of the oscillations vanishes. These points are called the nodes of the standing wave, Fig. (4).



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2. c

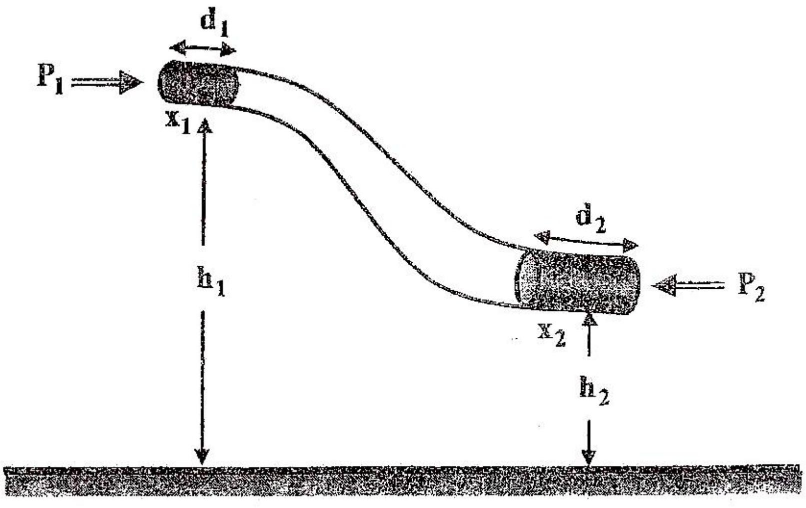

Figure (2)

Work done in moving a mass m_l of liquid a distance d_1 is

$$W_1 = F_1\, d_1 = P_1\, A_1\, \upsilon_1\, \Delta t$$

And the work done in moving a mass m_2 of liquid a distance d_2 is

$$W_2 = F_2\, d_2 = P_2\, A_2\, \upsilon_2\, \Delta t$$

But $A_1 \upsilon_1 = A_2 \upsilon_2$, so

$$W_2 = P_2\, A_1\, \upsilon_1\, \Delta t$$

The net work done on the liquid is

$$W = -(W_2 - W_1) = A_1\, \upsilon_1\, \Delta t (P_1 - P_2) \qquad (1)$$

This work done on the liquid contributes for the changes in kinetic energy and gravitational energy. Change in kinetic energy is

$$KE = \frac{1}{2} m_2 \upsilon_2^2 - \frac{1}{2} m_1 \upsilon_1^2$$

From the continuity equation $m_1 = m_2 = \rho A_1 \upsilon_1 \Delta t$ so

$$KE = \frac{1}{2}\rho A_1 \upsilon_1 \Delta t(\upsilon_2^2 - \upsilon_1^2) \qquad (2)$$

Change in gravitational energy is

$$PE = m_2 g h_2 - m_1 g h_1 \quad , \quad m_1 = m_2$$
$$= m_1 g(h_2 - h_1)$$

$$PE = \rho A_1 \upsilon_1 \Delta t\, g(h_2 - h_1) \qquad (3)$$

From the conservation low of energy, the net work done equal to the sum of the change in kinetic and gravitational energies.

$$W = KE + PE$$

From Eqs. (1), (2) and (3)

$$A_1\upsilon_1\Delta t(P_1 - P_2) = \frac{1}{2}\rho A_1\upsilon_1\Delta t(\upsilon_2^2 - \upsilon_1^2) + \rho A_1 \upsilon_1 \Delta t\, g(h_2 - h_1)$$
$$P_1 - P_2 = \frac{1}{2}\rho(\upsilon_2^2 - \upsilon_1^2) + \rho g(h_2 - h_1)$$
$$P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho g h_2$$

Which means that

$$P + \frac{1}{2}\rho\upsilon^2 + \rho g h = \text{const.} \qquad (4)$$