Benha University | **4st year student, First Term**
Faculty of science | **Final Exam.2015-2016**
Physics Department | **Time: 2 hours. Date: 4-1-2016**

Magnetic resonance and Mossbauer spectrum,(453phy)

Answer All questions: (80 Marks)

Part 1

1- a- Explain Mechanical (Sonic) resonance and show the conditions of occurrence of resonance. [7 Marks]

b- Explain Nuclear Gamma Resonance and show the nuclear reaction for the excited source, [7 Marks]

2-a- Explain the history of discovering Mossbauer phenomena and show the importance of research of Mossbauer spectroscopy. [7Marks]

b- Calculate the energy required to excite Na^{23}_{11} and occurrence of yellow color ($\lambda = 5890$). [7Marks]

3- a-Write about one : Recoil energy shift or Doppler broadening. [7Marks]

b- Excited source of intensity 26 m Curie with f=0.6. calculate the photons number of Gamma rays by Bequrel to resonance. [5Marks]

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Part 2

1. Proof that diamagnetic susceptibility is Negative and Independent of temperature. [5 Marks]
2. Derive an expression for Curie's Law. [5 Marks]
3. Give short note about Brillouin & Bloch function and Lamor relationship. [5 Marks]
4. Give Quantum mechanical (QM) description of a spin in a magnetic field. [5 Marks]
5. Use Rotating Frame of reference to solve the motion of magnetic moment in the general case of a time varying B1(t). [10 Marks]
6. Sketch stages of NMR echo generation. [10 Marks]

Prof. Fathy Salman& Dr. Eslam Sheha

Part 2 Answer

$$I = \text{charge}/\text{time} = e\omega/(2\pi),$$

so there is a magnetic moment

$$\mu = IA = e\omega a^2/2.$$

- The electron is kept in this orbit by a central force

$$F = m_e\omega^2 a.$$

- Now if a flux density $\mathcal{B}$ is applied in the z direction there will be a Lorentz force giving an additional force along a radius

$$\Delta F = ev\mathcal{B} = e\omega a\mathcal{B}$$

- If we assume the charge keeps moving in a circle of the same radius it will have a new angular velocity ω',

$$m_e\omega'^2 a = F - \Delta F$$

so

$$m_e\omega'^2 a = m_e\omega^2 a - e\omega a\mathcal{B},$$

or

$$\omega'^2 - \omega^2 = -\frac{e\omega\mathcal{B}}{m_e}.$$

- If the change in frequency is small we have

$$\omega'^2 - \omega^2 \approx 2\omega\Delta\omega,$$

where $\Delta\omega = \omega' - \omega$. Thus

$$\Delta\omega = -\frac{e\mathcal{B}}{2m_e}.$$

where $\frac{e\mathcal{B}}{2m_e}$ is called the *Larmor frequency*.

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- Substituting back into
$$\mu = IA = e\omega a^2/2,$$
we find a change in magnetic moment
$$\Delta\mu = -\frac{e^2 a^2}{4m_e}\mathcal{B}.$$

- Recall that a was the radius of a ring of current perpendicular to the field: if we average over a spherical atom
$$a^2 = \langle x^2\rangle + \langle y^2\rangle = \frac{2}{3}\left[\langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle\right] = \frac{2}{3}\langle r^2\rangle,$$
so
$$\Delta\mu = \frac{e^2\langle r^2\rangle}{6m_e}\mathcal{B},$$

- If we have n atoms per volume, each with p electrons in the outer shells, the magnetisation will be
$$\mathcal{M} = np\Delta\mu,$$
and
$$\chi = \frac{\mathcal{M}}{\mathcal{H}} = \mu_0\frac{\mathcal{M}}{\mathcal{B}} = -\frac{\mu_0 npe^2\langle r^2\rangle}{6m_e}.$$

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- An atomic angular momentum J, made of spin S and orbital angular momentum quantum number L, will have a magnetic moment $g_J\mu_B J$, where g_J is the Landé g-factor
$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}.$$

- If we write $x = g_J\mu_B\mathcal{B}/k_BT$, the average atomic magnetic moment will be
$$\langle\mu\rangle = \frac{\sum_{m=-J}^{J} mg_J\mu_B e^{mx}}{\sum_{m=-J}^{J} e^{mx}}.$$

- If we assume that T is large and/or $\mathcal{B}$ is small, we can expand the exponential, giving
$$\langle\mu\rangle \approx g_J\mu_B\frac{\sum_{m=-J}^{J} m(1+mx)}{\sum_{m=-J}^{J}(1+mx)}.$$

- We can evaluate this if we note that

$$
\begin{aligned}
\sum_{m=-J}^{J} 1 &= 2J+1 \\
\sum_{m=-J}^{J} m &= 0 \\
\sum_{m=-J}^{J} m^2 &= \frac{1}{3} J(J+1)(2J+1)
\end{aligned}
$$

then

$$
\begin{aligned}
\langle \mu \rangle &\approx g_J \mu_{\mathrm{B}} \frac{x J(J+1)(2J+1)}{3(2J+1)} \\
&= \frac{g_J^2 \mu_{\mathrm{B}}^2 \mathcal{B} J(J+1)}{3 k_{\mathrm{B}} T},
\end{aligned}
$$

- This leads to a susceptibility

$$
\chi = \frac{\mu_0 n g_J^2 \mu_{\mathrm{B}}^2 J(J+1)}{3 k_{\mathrm{B}} T.}
$$

- This is *Curie's Law*, often written

$$
\chi = \frac{C}{T}.
$$

Brillouin function [edit]

The **Brillouin function**[1][2] is a special function defined by the following equation:

$$B_J(x) = \frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J}\coth\left(\frac{1}{2J}x\right)$$

The function is usually applied (see below) in the context where *x* is a real variable and *J* is a positive integer or half-integer. In this case, the function varies from -1 to 1, approaching +1 as $x \to +\infty$ and -1 as $x \to -\infty$.

The function is best known for arising in the calculation of the magnetization of an ideal paramagnet. In particular, it describes the dependency of the magnetization M on the applied magnetic field B and the total angular momentum quantum number J of the microscopic magnetic moments of the material. The magnetization is given by:[1]

$$M = Ng\mu_B J \cdot B_J(x)$$

where

- N is the number of atoms per unit volume,
- g the g-factor,
- μ_B the Bohr magneton,
- x is the ratio of the Zeeman energy of the magnetic moment in the external field to the thermal energy $k_B T$:

  $$x = \frac{g\mu_B J B}{k_B T}$$

- k_B is the Boltzmann constant and T the temperature.

Note that in the SI system of units B given in Tesla stands for the magnetic field, $B = \mu_0 H$, where H is the auxiliary magnetic field given in A/m and μ_0 is the permeability of vacuum.

Bloch equations

From Wikipedia, the free encyclopedia

For the wavefunction of a particle in a periodic potential, see Bloch wave.

In physics and chemistry, specifically in nuclear magnetic resonance (NMR), magnetic resonance imaging (MRI), and electron spin resonance (ESR), the **Bloch equations** are a set of macroscopic equations that are used to calculate the nuclear magnetization **M** = (M_x, M_y, M_z) as a function of time when relaxation times T_1 and T_2 are present. These are phenomenological equations that were introduced by Felix Bloch in 1946.[1] Sometimes they are called the equations of motion of nuclear magnetization. They are analogous to the Maxwell-Bloch equations.

Bloch equations in laboratory (stationary) frame of reference [edit]

Let **M**(*t*) = ($M_x(t)$, $M_y(t)$, $M_z(t)$) be the nuclear magnetization. Then the Bloch equations read:

$$\frac{dM_x(t)}{dt} = \gamma(\mathbf{M}(t) \times \mathbf{B}(t))_x - \frac{M_x(t)}{T_2}$$
$$\frac{dM_y(t)}{dt} = \gamma(\mathbf{M}(t) \times \mathbf{B}(t))_y - \frac{M_y(t)}{T_2}$$
$$\frac{dM_z(t)}{dt} = \gamma(\mathbf{M}(t) \times \mathbf{B}(t))_z - \frac{M_z(t) - M_0}{T_1}$$

where γ is the gyromagnetic ratio and **B**(*t*) = ($B_x(t)$, $B_y(t)$, $B_0 + \Delta B_z(t)$) is the magnetic field experienced by the nuclei. The *z* component of the magnetic field **B** is sometimes composed of two terms:

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- one, B_0, is constant in time,
- the other one, ΔB_z(t), may be time dependent. It is present in magnetic resonance imaging and helps with the spatial decoding of the NMR signal.

M(*t*) × **B**(*t*) is the cross product of these two vectors. M_0 is the steady state nuclear magnetization (that is, for example, when t → ∞); it is in the *z* direction.

Larmor precession

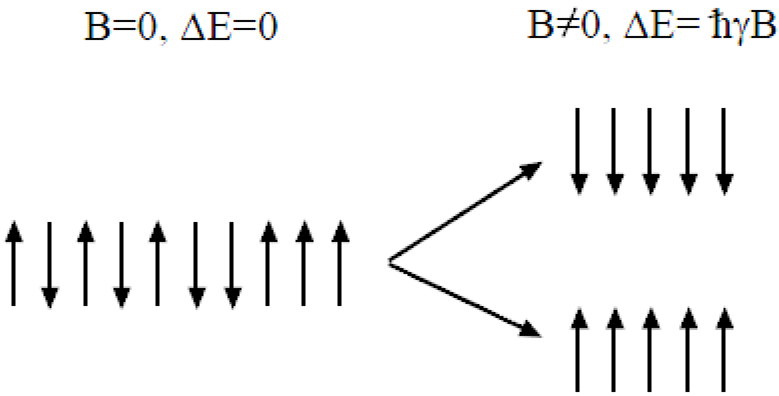
From Wikipedia, the free encyclopedia

In physics, **Larmor precession** (named after Joseph Larmor) is the precession of the magnetic moment of any object with a magnetic moment about an external magnetic field. Objects with magnetic moments have angular momentum and internal currents of electric charge related to their angular momentum; these include electrons, protons, other fermions, many atomic and nuclear systems, as well as classical macroscopic systems. The magnetic field exerts a torque on the magnetic moment,

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \gamma \vec{J} \times \vec{B}$$

where $\vec{\tau}$ is the torque, $\vec{\mu}$ is the magnetic dipole moment, $\vec{J}$ is the angular momentum vector, $\vec{B}$ is the external magnetic field, $\times$ symbolizes the cross product, and γ is the gyromagnetic ratio which gives the proportionality constant between the magnetic moment and the angular momentum. The phenomenon is similar to the precession of a tilted classical gyroscope in an external gravitational field.

With no applied magnetic field, all spins are in the same energy state (E=0). Their magnetic moments are randomly oriented are do not form any coherent magnetization. When placed in an applied magnetic field, the spin will tend to align with or opposite to the direction of applied magnetic field. These two states are known as "spin up" and "spin down," respectively. The spin-up state (in alignment) is slightly preferred, and thus has a lower energy level. The spin-down state is at a higher energy. A spin-up nuclei can absorb energy and transition to a spin-



The energy difference between these states is determined by the strength of the applied magnetic field, which will we will call B_0, in the following relationship:

$$\Delta E = \hbar\gamma B_0 = \hbar\omega_0 = hf_0$$

where γ is the gyromagnetic ratio, h is Plank's constant (h = 6.63e-34 J s = 4.14e-15 eV s) and $\hbar = h/2\pi$.

These are governed by thermal equilibrium condition, which are characterized by the Boltzmann distribution. Letting N+ be the higher energy state (spin-down) and N- be the lower energy state, Boltzmann dictates that:

$$\frac{N+}{N-} = e^{-\Delta E/kT}$$

where

k = Boltzmann's constant (8.62e-5 eV/K or 1.38e-23 J/K)

T = temperature (human body temperature = 310 K)

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$\Delta E = \hbar\gamma B_0$

In general, the exponent is extremely small and N+ and N- are nearly the same and approximately ½ of the total number of nuclei. Using the first two terms of the Taylor series expansion of the exponent, we get:

$$\frac{N+}{N-} \approx 1 - \frac{\Delta E}{kT}$$

$$\Delta N = (N-) - (N+) = \frac{\Delta E}{kT} N+ \approx \frac{\Delta E}{kT} \tfrac{1}{2} N_T$$

$$\Delta N = \frac{\hbar\gamma B_0}{kT} \tfrac{1}{2} N_T$$

For a rotation frame at ω_0, the coordinate axes are transformed in this way:

$$\mathbf{i}' = \mathbf{i}\cos(\omega_0 t) - \mathbf{j}\sin(\omega_0 t)$$

$$\mathbf{j}' = \mathbf{i}\sin(\omega_0 t) + \mathbf{j}\cos(\omega_0 t)$$

$$\mathbf{k}' = \mathbf{k}$$

Thus, when $\mathbf{B} = B_0\mathbf{k}$, the apparent $\mathbf{B}$ in the rotating frame is:

$$\mathbf{B}_{\mathbf{eff}} = (B_0 - \frac{\omega_{frame}}{\gamma})\mathbf{k} = (B_0 - \frac{\omega_0}{\gamma})\mathbf{k} = (B_0 - B_0)\mathbf{k} = 0$$

The x-y components of the magnetization are then:

$$m_{xy.rot}(t) = m_{xy}(t)\ \exp(i\ \omega_0\ t) = m_0$$

which is stationary. We now have a simple conversion of magnetization in the rotating frame and the lab frame. If $\mathbf{M} = [m_x, m_y, m_z]$ and $\mathbf{M}_{\mathbf{rot}} = [m_{x,rot}, m_{y,rot}, m_{z,rot}]$, then

$$m_{xy.rot} = m_{x,rot} + i\ m_{y,rot} = m_{xy}\ \exp(i\ \omega_0\ t)$$

$$m_{z,rot} = m_z$$

Let's now consider $\mathbf{B}(t) = [B_0 + \Delta B(t)]\mathbf{k}$. Here the magnetization in the rotation frame will appear to be precessing at

$$\omega_{rot}(t) = \gamma[B_0 + \Delta B(t)] - \omega_0 = \gamma \Delta B(t)$$

Thus, the apparent $\mathbf{B}$ in the rotating frame ($\omega_{\text{frame}}=\omega_0$) is:

$$\mathbf{B}_{\mathbf{eff}} = \mathbf{B} - \frac{\omega_0}{\gamma}\mathbf{k} = (B_0 + \Delta B(t) - B_0)\mathbf{k} = \Delta B(t)\mathbf{k}$$

The direction that the $\mathbf{M}_{\mathbf{rot}}$ points is given by the time integral of this frequency function:

$$\phi_{rot}(t) = \gamma \int_0^t \Delta B(\tau) d\tau$$

And thus,

$$m_{xy,rot}(t) = m_0 e^{-i\left[\gamma \int_0^t \Delta B(\tau) d\tau\right]}$$

The Bloch equation can now be rewritten for use in the rotating frame:

$$\frac{d\mathbf{M}_{rot}}{dt} = \mathbf{M}_{rot} \times \gamma \mathbf{B}_{eff}$$

where **M** can be derived from $\mathbf{M}_{rot}$ using:

$$m_{xy} = m_{xy,rot} \exp(-i\ \omega_0\ t)$$

$$m_z = m_{z,rot}$$

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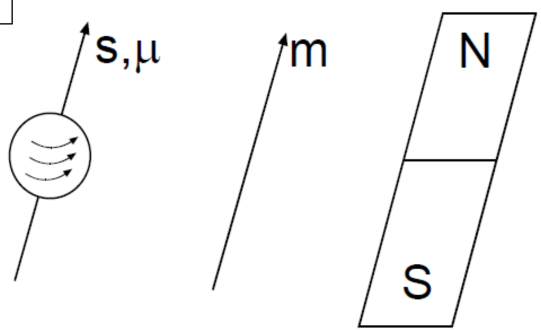

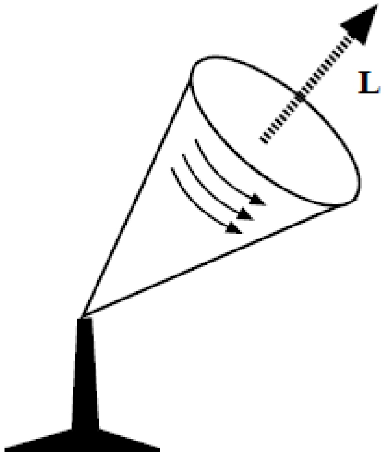
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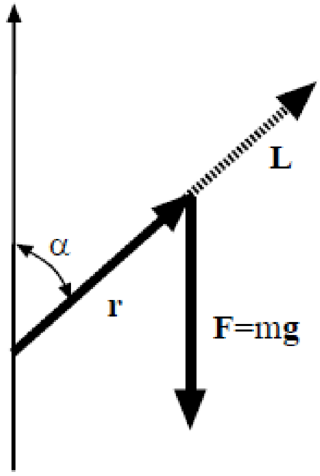
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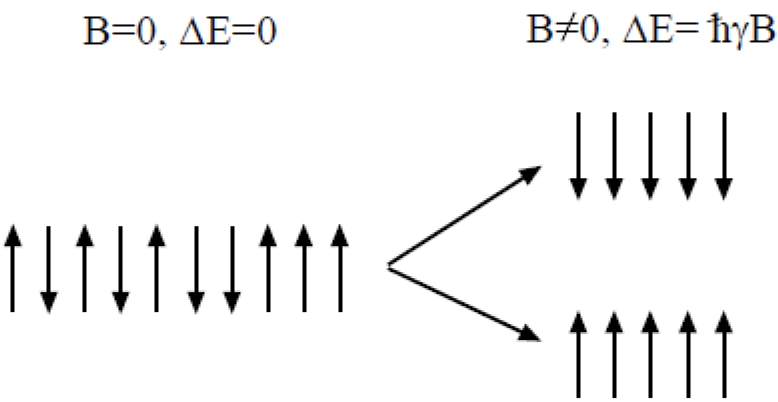
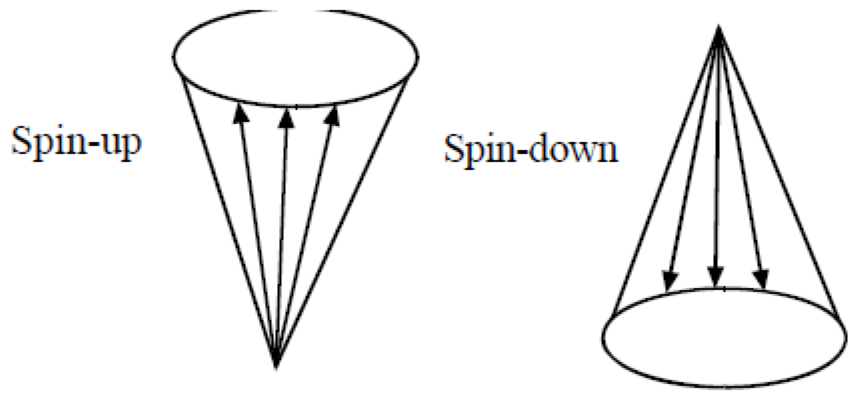
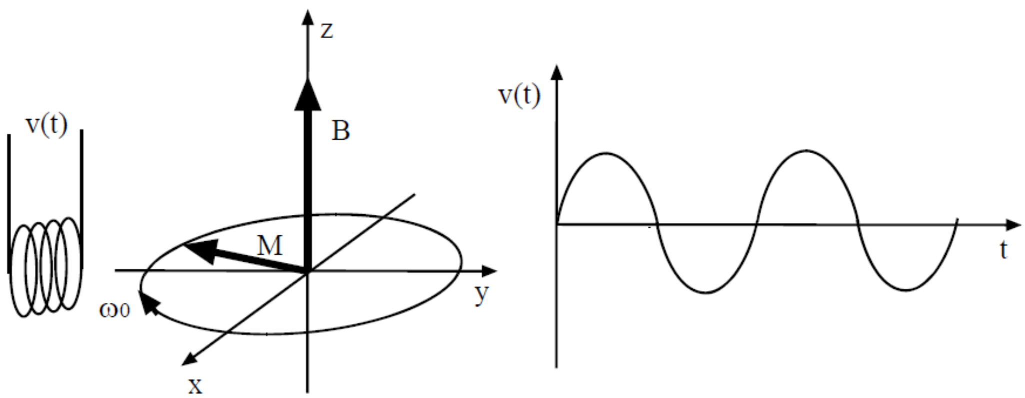

B=0, ΔE=0
B≠0, ΔE= ħγB


Spin-up
Spin-down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