جامعة بنـها – كلية العلوم – قسم الرياضيات

لطلاب المستوى الثالث

يوم الامتحان: الاثنين ٤ / ١ / ٢٠١٦ م

المادة : رياضيات متقطعة (٣١٢ ر)

الممتحن: د . / محمد السيد عبدالعال عبدالغنى

مدرس بقسم الرياضيات بكلية العلوم

اسئله + نموذج إجابه

ورقة كاملة

رياضيات متقطعة (312 ر) لطلاب المستوى الثالث

أجب على الاسئله التاليه (الدرجة الكلية ١٢٠ درجة)

Question 1. **السؤال الأول (٣٠ درجة) :-**

1- Let A , B , C, D be sets, **prove that:**

I. $A \subseteq B$ **if and only if** $P(A) \subseteq P(B)$

II. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

2- Let $f : A \to B$ and $g : B \to C$ be two functions **prove that**: **if** *f* and *g* are both bijection **then** so, too is $g \circ f$.

3- For all propositions *p, q, r,* **Prove that:**

I. $(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv \overline{p}$

II. $[(p \to q) \wedge (p \vee r)] \Rightarrow (q \vee r)$.

Question 2. **السؤال الثانى (٣٠ درجة) :-**

1. **Define** *the complete graph* K_n , *the complete bipartite graph* $K_{r,s}$ and *Eulerian path*, and for **which values** of *n, r, s,* the graphs K_n , $K_{r,s}$ are **Eulerian**?

2. **Show** the following function is a **bijection** and find its **inverse**:
$f : R - \{5\} \to R - \{2\},\ f(x) = \dfrac{2x+1}{x-5}\ \forall x \in R - \{5\}$.

3. A relation $\equiv_5$ on the set *Z* is defined by $a \equiv_5 b$ if and only if $a - b = 5k$ for some $k \in Z$, show that $\equiv_5$ is an equivalent relation and describe the equivalence classes [3], [-1].

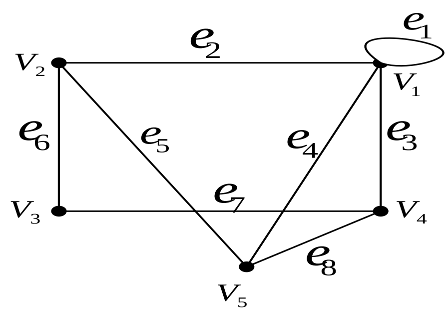
Question 3. **السؤال الثالث (٣٠ درجة) :-**

1. Let $f : Q \to Q$ be a bijection function $f(x) = 2x + 1\ \forall x \in Q$. Find $f(Z^+)$, $f^{-1}(Z^+)$.

2. **Design** a logic network for the following so that the output is described by the Boolean expression given: $x_1 x_3 \oplus \overline{x_1} \oplus x_2 \overline{x_3}$.

انظر خلف الورقة

3. **Find** the matrix A^3, where A be the adjacency matrix, for the following graph: and **write** all edge sequences of length 3 joining v_2, v_3.



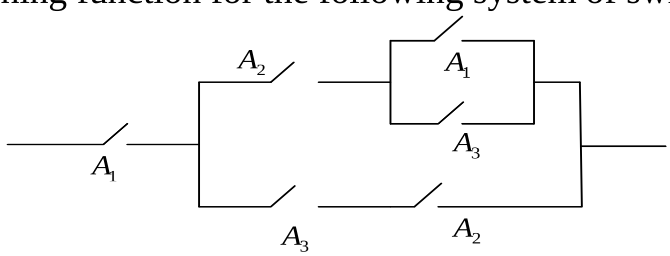
Question 4. **السؤال الرابع (30 درجة) :-**

1. Let *S* be a non-empty set and consider *P*(*S*), the power set of *S*, together with the binary operations of union and intersection and the operation of complementation then:

   I. **prove that** $(P(S), \cup, \cap, \bar{\ }, \phi, S)$ is a Boolean algebra.

   II. Given $A \in P(S)$, **prove that** there is only one $\bar{A} \in P(S)$ such that $A \cup \bar{A} = S$ and $A \cap \bar{A} = \phi$.

2. **Define** a switching function for the following system of switches:



3. Let *f*, *g*, and *h* be functions $R \to R$ defined respectively by

$$f(x) = 2x + 1, \quad g(x) = \frac{1}{x^2 + 1}, \text{ and } h(x) = \sqrt{x^2 + 1}.$$

Find expressions for $(f \circ (g \circ h))(x)$.

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Good Luck ! انتهت أسئلة

مع أطيب تمنياتى بالتوفيق و النجاح

نموذج اجابه لأمتحان رياضيات متقطعة (٣١٢ ر) لطلاب المستوى الثالث

(الدرجة الكلية ١٢٠ درجة)

اجابة السؤال الأول (30 درجة) :-

1- Let A , B , C, D be sets, prove that:

I. $A \subseteq B$ **if and only if** $P(A) \subseteq P(B)$

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1- $A \subseteq B$ **if and only if** $P(A) \subseteq P(B)$

To prove the biconditional statement we prove the two conditional statements: $A \subseteq B \Rightarrow P(A) \subseteq P(B)$ and $P(A) \subseteq P(B) \Rightarrow A \subseteq B$.

Firstly, suppose $A \subseteq B$. We must show that $P(A) \subseteq P(B)$, so let $X \in P(A)$. This means $X \subseteq A$. Since $A \subseteq B$, it follows that $X \subseteq B$, which means that $X \in P(B)$. Since $X \in P(A)$ implies $X \in P(B)$, we conclude that $P(A) \subseteq P(B)$, which completes the first half of the proof. To prove the converse statement, suppose $P(A) \subseteq P(B)$. Since $A \in P(A)$, it follows that $A \in P(B)$. This means that $A \subseteq B$, which completes the proof.

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II- $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let $(a, x) \in A \times (B \cap C)$. By the definition of the Cartesian product, this means that $a \in A$ and $x \in (B \cap C)$. Thus $x \in X$, so (a, x) belongs to $A \times B$; and $x \in C$, so (a, x) belongs to $A \times C$ as well. Therefore $(a, x) \in (A \times B) \cap (A \times C)$, which proves that $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$.

To prove the subset relation the other way round as well, let $(a, x) \in (A \times B) \cap (A \times C)$.

Then $(a, x) \in (A \times B)$, so $a \in A$ and $x \in B$; and $(a, x) \in (A \times C)$, so $a \in A$ and $x \in C$. Therefore $a \in A$ and $x \in (B \cap C)$ which means that the ordered pair (a, x) belongs to the Cartesian product $A \times (B \cap C)$. Hence $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$.

The conclusion that the sets $A \times (B \cap C)$ and $(A \times B) \cap (A \times C)$ are equal now

2- Let $f: A \to B$ and $g: B \to C$ be two functions **prove that**: **if** *f* and *g* are both bijection **then** so, too is $g \circ f$.

(i) Suppose *f* and *g* are injections. Let a, $a_1 \in$ A, b = f (a) and b_1 = f (a_1).

Then $g \circ f(a) = g \circ f(a_1)$

$g(f(a)) = g(f(a_1)$

$\Rightarrow g(b) = g(b_1) \Rightarrow b = b_1 \Rightarrow$ (since g is injective)

$f(a) = f(a_1)$ (since $f(a) = b, f(a_1) = b_1$

$\Rightarrow a = a_1 \Rightarrow$ (since f is injective).

Hence $g \circ f$ is an injection.

(ii) Suppose f and g are surjections and let $c \in C$. Since g is surjective, there exists $b \in B$ such that $g(b) = c$, and since f is surjective, there exists $a \in A$ such that $f(a) = b$. Therefore there exists $a \in A$ such that

$g \circ f(a) = g(f(a)) = g(b) = c$

so g ◦ f is surjective.

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3- For all propositions *p, q, r,* **Prove that:**

I. $(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv \overline{p}$

II. $[(p \to q) \wedge (p \vee r)] \Rightarrow (q \vee r)$.

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$$(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv (\overline{p} \wedge q) \vee (\overline{p} \wedge \overline{q}) \equiv (\overline{p} \wedge (\overline{q} \vee q) \equiv \overline{p} \wedge t \equiv \overline{p}$$

ii- $[(p \to q) \wedge (p \vee r)] \Rightarrow (q \vee r)$

p	q	r	$p \to q$	$p \vee r$	$(p \to q) \wedge (p \vee r)$	$q \vee r$	$[(p \to q) \wedge (p \vee r)] \to (q \vee r)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	0	1	0	1	0	1	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	1

0	0	1	1	1	1	1	1
0	0	0	1	0	0	0	1

أجابة السؤال الثانى (30 درجة) :-

1. **Define** *the complete graph* K_n , *the complete bipartite graph* $K_{r,s}$ and *Eulerian path*, and for **which values** of *n, r, s*, the graphs K_n , $K_{r,s}$ are **Eulerian**?

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A complete graph is a simple graph in which every pair of distinct vertices is joined by an edge.
A complete bipartite graph is a bipartite graph such that every vertex of V_1 is joined to every vertex of V_2 by a unique edge.
An Eulerian path in a graph G is a closed path which includes every edge of G . A graph is said to be Eulerian if it has at least one Eulerian path.
The complete graph K_n is (n−1)-regular−every vertex has degree n−1. Since it is connected, K_n is Eulerian if and only if n is odd (so that n − 1is even).
A complete bipartite graph $K_{r,s}$ is Eulerian if and only if *r,s* is even.

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2. **Show** the following function is a **bijection** and find its **inverse**:

$$f : R-\{5\} \to R-\{2\},\ f(x) = \frac{2x+1}{x-5}\ \forall x \in R-\{5\}.$$

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To show that *f* is an injection we prove that, for all real numbers *x* and *y*, *f (x) = f (y)* implies *x = y*. Now *f (x) = f (y)*

$$\Rightarrow \frac{2x+1}{x-5} = \frac{2y+1}{y-5}$$

easily $\Rightarrow$ $x = y$ so *f* is injective.
To show that *f* is a surjection, let *y* be any element of the codomain *f* . We need to find x ∈ R-{5} such that f (x) = y. Let $x = \frac{1+5y}{y-2}$. Then *x* ∈ *R-{5}* and

$$f(x) = [2\frac{1+5y}{y-2}+1] \div [\frac{1+5y}{y-2}-5] = \frac{2+10y+y-2}{y-2} \div \frac{1+5y-5y+10}{y-2} = \frac{11y}{11} = y$$

so *f* is surjective.

To find f^{-1} we simply use its definition: if y = f (x) then $x = f^{-1}(y)$.
Now

$$y = f(x) \Rightarrow y = \frac{2x+1}{x-5} \Rightarrow x = \frac{1+5y}{y-2}$$

$x = f^{-1}(y) = \frac{1+5y}{y-2}$.

Therefore the inverse function is $f^{-1} : R-\{2\} \to R-\{5\}, f^{-1}(y) = \frac{1+5y}{y-2}$.

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3. A relation $\equiv_5$ on the set Z is defined by $a \equiv_5 b$ if and only if $a-b=5k$ for some $k \in Z$, show that $\equiv_5$ is an equivalent relation and describe the equivalence classes [3], [-1].

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In this case $a \equiv_5 b$ if and only if a −b = 5k for some integer k; that is, if and only if there exists an integer k such that a = 5k + b.

Firstly, R is reflexive since a-a = 5 0,

Secondly, if $a \equiv_5 b$ i.e. a-b =5k then b-a =−5k so implies $b \equiv_5 a$ therefore $\equiv_5$ is symmetric.

Thirdly, suppose $a \equiv_5 b$ and $b \equiv_5 c$; then there exist integers k such that a-b =5k and b-c =5k_1.

Combining these two equations gives a-c =5(k-k_1) therefore $a \equiv_5 c$

where (k-k1)is an integer. Thus $a \equiv_5 b$ and $b \equiv_5 c$ implies $a \equiv_5 c$ so $\equiv_5$ is transitive.

Therefore [p] = {q ∈ z : q = 5k + p, for some k ∈ z}.

[3] = {q ∈ z : q = 5k + 3, for some k ∈ z}.

[-1] = {q ∈ z : q = 5k + -1, for some k ∈ z}.

أجابة السؤال الثالث (30 درجة) :-

1. Let $f: Q \to Q$ be a bijection function $f(x) = 2x+1 \; \forall x \in Q$.

   Find $f(Z^+)$, $f^{-1}(Z^+)$.

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This function can be represented visually by a modified version of the ‘arrow diagram’ −

Z^{+}: 1 2 3 4 5 6…..

f |

: 3 5 7 9 11 13….. $f(Z^+)$

$$f(Z^+) = \{y = 2x+1, x \in z^+$$

$$f^{-1}(Z^+) = \{x = \frac{y-1}{2}, y \in z^+\}$$

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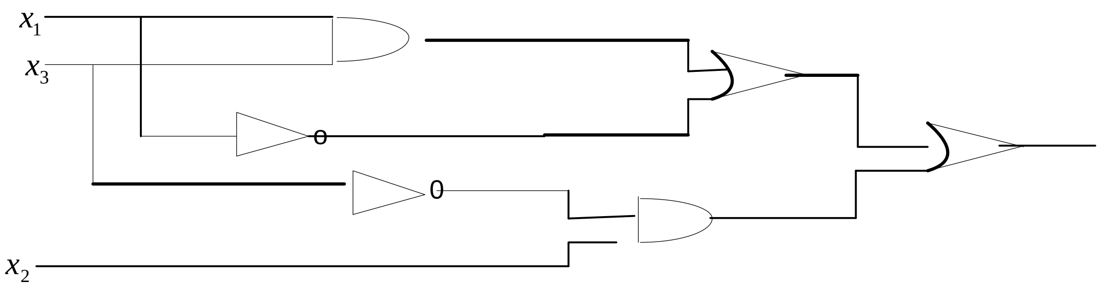
2. Design a logic network for the following so that the output is described by the Boolean expression given: $x_1 x_3 \oplus \overline{x_1} \oplus x_2 \overline{x_3}$.



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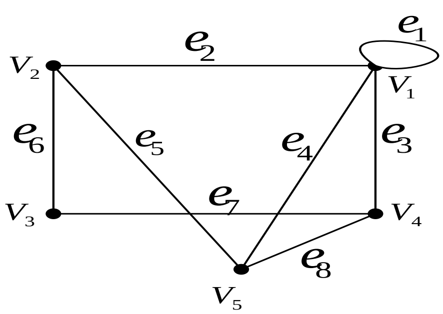

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$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}, \quad A^3 = \begin{pmatrix} 11 & 9 & 4 & 8 & 9 \\ 9 & 4 & \underline{\underline{6}} & 8 & 3 \\ 4 & 5 & 0 & 2 & 5 \\ 8 & 8 & 2 & 8 & 8 \\ 9 & 3 & 6 & 8 & 3 \end{pmatrix}$$

$e_6 e_6 e_6$; $e_5 e_8 e_7$; $e_2 e_1 e_2$; $e_6 e_7 e_7$; $e_5 e_5 e_6$; $e_2 e_2 e_6$;

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أجابة السؤال الرابع (٣٠ درجة) :

1. Let S be a non-empty set and consider $P(S)$, the power set of S, together with the binary operations of union and intersection and the operation of complementation then:

   I. **prove that** $(P(S), \cup, \cap, \bar{\ }, \phi, S)$ is a Boolean algebra.

   II. Given $A \in P(S)$, **prove that** there is only one $\overline{A} \in P(S)$ such that $A \cup \overline{A} = S$ and $A \cap \overline{A} = \phi$.

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Let S be a non-empty set and consider P(S), the power set of S, together with the binary operations of union and intersection and the operation of complementation, where, for all $A \in P$(S), $\overline{A} = S - A$.

(a) the operations ∪ and ∩ are associative;

(b) the operations ∪ and ∩ are commutative;

(c) the operation ∪ is distributive over ∩ and ∩ is distributive over ∪;

(d) the sets φ and S belong to P(S) and

$A \cup \varphi = \varphi \cup A = A$

$A \cap S = S \cap A = A$

for all A ∈ P(S). Thus φ and S are the identities for ∪ and ∩ respectively;

(e) for any A ∈ P(S), .A ∈ P(S) and A ∪ $\overline{A}$ = S and A ∩ $\overline{A}$ = φ.

Since these are precisely the axioms B1–B5 we can conclude that (P(S),∪,∩, -, φ., S) is a Boolean algebra. The sum and product operations are union and intersection respectively, and we can write 0 = φ. and 1 = S for the two identities.

I. Given $A \in P(S)$, **prove that** there is only one $\overline{A} \in P(S)$ such that $A \cup \overline{A} = S$ and $A \cap \overline{A} = \phi$.

Suppose that .b1 and .b2 are both complements of an element b of a Boolean algebra (P(S),∪,∩, -, φ., S) = (B,⊕, ∗, -, 0, 1). This means that

$b \oplus .b_1 = .b_1 \oplus b = 1$, $b \oplus .b_2 = .b_2 \oplus b = 1$

$b * .b_1 = .b_1 * b = 0$, $b * .b_2 = .b_2 * b = 0.$, $.b_1 = \overline{b_1}$

Thus we have

.b1 = .b1 ∗ 1 (axiom B1)

= .b1 ∗ (b ⊕ .b2)

= (.b1 ∗ b) ⊕ (.b1 ∗ .b2) (axiom B4)

= 0 ⊕ (.b1 ∗ .b2)

= 0 ⊕ (.b2 ∗ .b1) (axiom B3)

= (.b2 ∗ b) ⊕ (.b2 ∗ .b1)

= .b2 ∗ (b ⊕ .b1) (axiom B4)

= .b2 ∗ 1

= .b2 (axiom B1).

We have shown that .b1 = .b2 and so we can conclude that the complement is unique.

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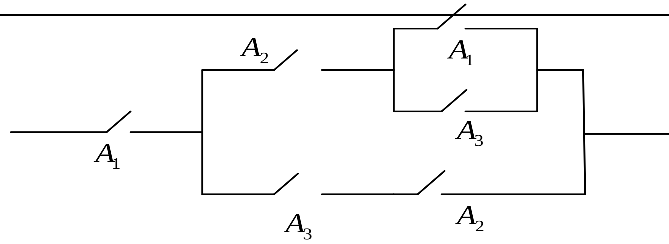
2. **Define** a switching function for the following system of switches:



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$f(x_1, x_2, x_3) = x_1[x_2(x_1 \oplus x_3) \oplus x_3 x_2]$

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3. Let *f*, *g*, and *h* be functions $R \rightarrow R$ defined respectively by

$$f(x) = 2x + 1, \quad g(x) = \frac{1}{x^2 + 1}, \text{ and } h(x) = \sqrt{x^2 + 1}.$$

Find expressions for $(f \circ (g \circ h))(x)$.

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$$(f \circ (g \circ h))(x) = f(g(h(x))) = f(g(\sqrt{x^2 + 1})) = f(\frac{1}{(\sqrt{x^2 + 1})^2 + 1})$$

$$= f(\frac{1}{x^2 + 2}) = 2\frac{1}{x^2 + 2} + 1 = \frac{x^2 + 4}{x^2 + 2}.$$

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