Benha University **Faculty of Sience**

Final Exam **Date**: 28/12/2016
Class: 4 Year Students **Time:** 3 hours
Subject: Scientific Computing 1 **Examiner:** Dr. El-Sayed Badr

Answer the following questions:

Q1: (16 points)

a) Define the following terms:

Partially ordered sets:

A partially ordered set or poset is a pair (X; P) where X is a set and P is a reflexive, antisymmetric, and transitive binary relation on X.

height of a poset :

The **height** of a poset (X; P), denoted height(P), is the largest h for which there exists a chain of *h* points in P.

width of a poset :

the width of a poset P = (X; P), denoted width(P), is the largest w for which there exists an antichain of w points in P.

jump number of a poset:

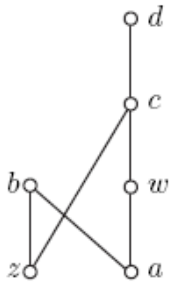
Let $L = [x_1, x_2, ..., x_n]$ be a linear extension of a poset ***P*** = (*X*, *P*)**.** We say that *L* has a jump at *i* if $x_i \| x_{i+1}$ in *P* and $x_i \prec x_{i+1}$ in L. The number of jumps of L is denoted by s(L, P). The jump number of ***P*** = (*X*, *P*) is defined by

s(**P**) = s (X, P) = min{ s (L, P): L is a linear extension of **P**}.

Moreover, if s (L, P) = s (P) then L is called an optimal linear extension.

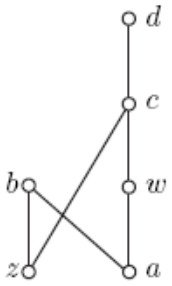
b) How to represent of the partially ordered sets and compare between the different represenation?

c) Find all the linear extension of the following poset :



Solution:

The poset shown in the following figure and has 11 linear extensions.



L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	L_{11}
d	d	d	b	d	d	d	b	d	d	b
c	c	b	d	c	c	b	d	c	b	d
w	b	c	c	w	b	c	c	b	c	c
b	w	w	w	b	w	w	w	z	z	z
a	a	a	a	z	z	z	z	w	w	w
z	z	z	z	a	a	a	a	a	a	a

Q2: (16 points)

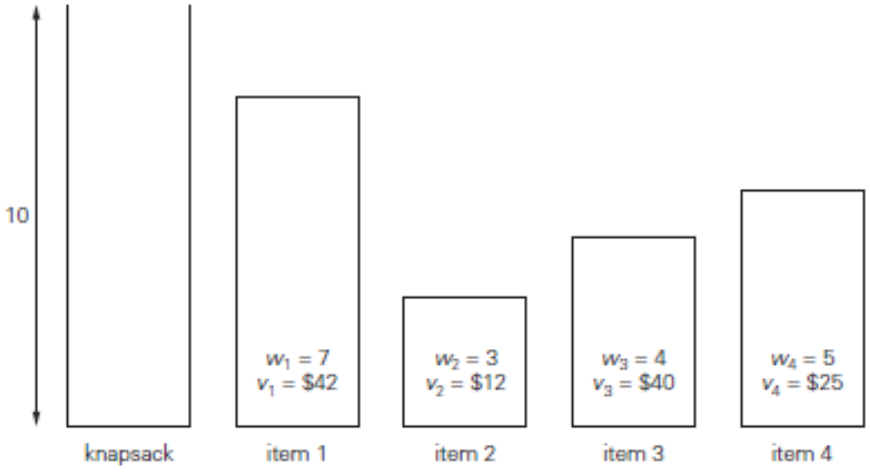
a) Solve the following Knapsack problem by three methos and compare among them :

Items	1	2	3	4
Weights	7	3	4	5
values	42	12	40	25

where the capacity of the Knapsack is 10 kg ?

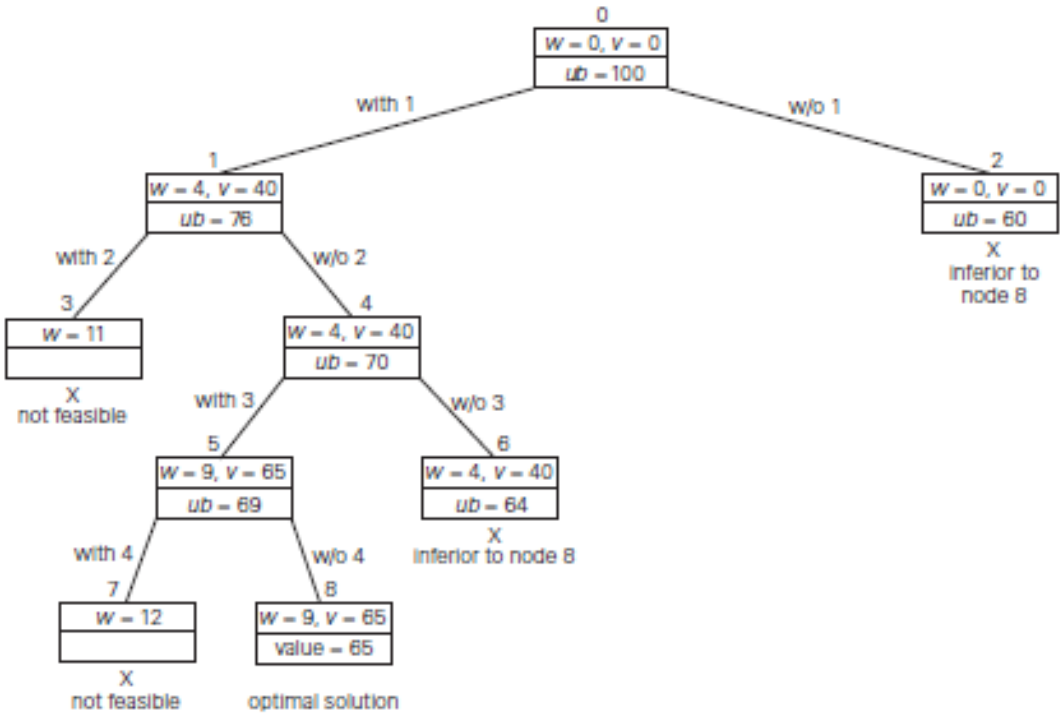
Solution:

First method : Brute force



Subset	Total weight	Total value
∅	0	$ 0
{1}	7	$42
{2}	3	$12
{3}	4	$40
{4}	5	$25
{1, 2}	10	$54
{1, 3}	11	not feasible
{1, 4}	12	not feasible
{2, 3}	7	$52
{2, 4}	8	$37
{3, 4}	**9**	**$65**
{1, 2, 3}	14	not feasible
{1, 2, 4}	15	not feasible
{1, 3, 4}	16	not feasible
{2, 3, 4}	12	not feasible
{1, 2, 3, 4}	19	not feasible

Second Method: Branch and bound



$$ub = v + (W - w)\left(\frac{v_{i+1}}{w_{i+1}}\right)$$

Third method: Linear programming

$$\max V = \sum_{i=1}^{n} v_i x_i$$

s.t

$$\sum_{i=1}^{n} w_i x_i \le W \quad ; x_i \ge 0$$

$$\max V = 20x_1 + 30x_2 + 66x_3 + 40x_4 + 60x_5$$

s.t.

$$10x_1 + 20x_2 + 30x_3 + 40x_4 + 50x_5 \ge 100$$

$$x_1 \ge 1$$

$$x_2 \ge 1$$

$$x_3 \ge 1$$

$$x_4 \ge 1$$

$$x_5 \ge 1$$

```
>> lb = zeros(5,1);
>> f=[-20;-30;-66;-40;-60];
>> b=[100;1;1;1;1;1];
>> A=[10 20 30 40 50;
      1 0 0 0 0;
      0 1 0 0 0;
      0 0 1 0 0;
      0 0 0 1 0;
      0 0 0 0 1];
>> [x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb)
x =
    1.0000
    1.0000
    1.0000
    0.0000
    0.8000
fval =  -164.0000
exitflag =

     1
output =
iterations: 7
```

b) What is the difference between the polynomial algorithm and the exponential algorithm (using an example) ?

Q3: (16 points)

a) Name and describe the main features/elements/steps of Genetic Algorithms (GA) ?

Solution:

Answer: *Genetic Algorithms (GA) use principles of natural evolution. There are five important features of GA:*

Encoding *possible solutions of a problem are considered as individuals in a population. If the solutions can be divided into a series of small steps (building blocks), then these steps are represented by genes and a series of genes (a chromosome) will encode the whole solution. This way different solutions of a problem are represented in GA as chromosomes of individuals.*

Fitness Function *represents the main requirements of the desired solution of a problem (i.e. cheapest price, shortest route, most compact arrangement, etc). This function calculates and returns the fitness of an individual solution.*

Selection *operator defines the way individuals in the current population are selected for reproduction. There are many strategies for that (e.g. roulette–wheel, ranked, tournament selection, etc), but usually the individuals which are more fit are selected.*

Crossover *operator defines how chromosomes of parents are mixed in order to obtain genetic codes of their offspring (e.g. one–point, two–point, uniform crossover, etc). This operator implements the inheritance property (offspring inherit genes of their parents).*

Mutation *operator creates random changes in genetic codes of the offspring. This operator is needed to bring some random diversity into the genetic code. In some cases GA cannot find the optimal solution without mutation operator (local maximum problem).*

b) Suppose a genetic algorithm uses chromosomes of the form x = abcdefgh with a fixed length of eight genes. Each gene can be any digit between 0 and 9. Let the fitness of individual x be calculated as:

$$f(x) = (a + b) - (c + d) + (e + f) - (g + h),$$

and let the initial population consist of four individuals with the following chromosomes:

x1 = 6 5 4 1 3 5 3 2
x2 = 8 7 1 2 6 6 0 1
x3 = 2 3 9 2 1 2 8 5
x4 = 4 1 8 5 2 0 9 4

Evaluate the fitness of each individual,showing all your workings, and arrange them in order with the fittness first and the least fittness last ?

Solution:

$$
\begin{aligned}
f(x_1) &= (6+5)-(4+1)+(3+5)-(3+2)=9 \\
f(x_2) &= (8+7)-(1+2)+(6+6)-(0+1)=23 \\
f(x_3) &= (2+3)-(9+2)+(1+2)-(8+5)=-16 \\
f(x_4) &= (4+1)-(8+5)+(2+0)-(9+4)=-19
\end{aligned}
$$

The order is x_2, x_1, x_3 *and* x_4.

Good Luck