جامعة بنـها – كلية العلوم – قسم الرياضيات

لطلاب المستوى الثانى

يوم الامتحان: الاربعاء ١٨ / ١ / ٢٠١٧ م

المادة : رياضيات متقطعة (٢٢٥ ر)

الممتحن: د . / محمد السيد عبدالعال عبدالغنى

مدرس بقسم الرياضيات بكلية العلوم

اسئله + نموذج إجابه

ورقة كاملة

رياضيات متقطعة (٢٢٥ ر) لطلاب المستوى الثانى

Answer the following questions: (80 marks) أجب على الاسئله التاليه (الدرجة الكلية ٨٠ درجة)

Question 1. السؤال الأول (35 درجة) :-

1- A relation R on Z is defined by $m\,R\,n$ if and only if $\frac{m-n}{6} \in Z$. Show that R is an equivalence relation and describe the equivalence class of [-5].

2- Show that $[\overline{(p \leftrightarrow q)}]$ logically implies $[(p \leftrightarrow \bar{q})]$

3- Let $R = \{(a,a),(a,b),(a,c),(b,b),(b,c)\}$ be a relation on the set $\{a,b,c,d\}$. What is the minimum number of elements which need to be added to R in order that it becomes: (i) reflexive; (ii) symmetric; (iii) anti-symmetric; (iv) transitive ?

4- Let A , B , C are sets, **prove that:**

I. $A \cup (B - C) = (A \cup B) - (\bar{A} \cap C)$.

II. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Question 2. السؤال الثانى (25 درجة)

1. Let *f*, *g* and *h* be functions $f : R \to R$ defined respectively by :

   $f(x) = (5x-3), \quad g(x) = x^3, \quad h(x) = \sqrt{x^2+2}$

   i. **Find** expressions $((g \circ h) \circ f)(x)$;
   ii. **Show that** $(f \circ g)(x)$ is bijective and **find** its inverse
   iii. **Prove that** $\operatorname{Im}(h \circ f)(x) \subseteq \operatorname{Im} h$

2. **Design** a logic network for the following so that the output is described by the following Boolean expression: $(x_1 x_2 \oplus x_1 \oplus \overline{x_2})$.

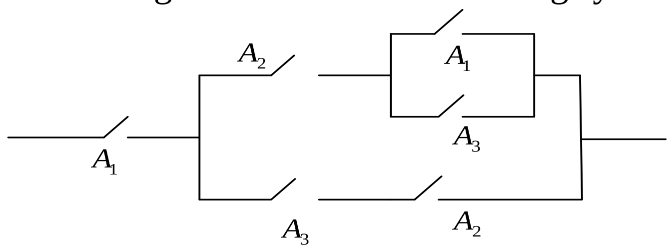
3. **Prove that** by definition, $A - (B \cap C) = (A - B) \cup (A - C)$.

انظر خلف الورقة

Question 3.

السؤال الثالث (25 درجة)

1. **Define** a switching function for the following system of switches:



2. Describe the degree sequence of :
   i. a null graph with *n* vertices;
   ii. The complete graph K_n;
   iii. An *r*-regular graph with *n* vertices
   iv. The complete bipartite graph $K_{n,m}$ where $n \le m$,

and **which values** of *n*, *r*, *m*, the graphs K_n, $K_{n,m}$ and *r*- regular graph are **Eulerian**?

3. **Define** a Boolean algebra $(B, \oplus, *, ^{-}, 0, 1)$ and for all $b_1, b_2 \in B$, **prove that**:
   $\overline{(b_1 * b_2)} = \overline{b_1} \oplus \overline{b_2}$.

انتهت أسئلة

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مع أطيب تمنياتى بالتوفيق و النجاح
د. محمد السيد عبدالعال

Good Luck !

نموذج اجابه لأمتحان رياضيات متقطعة (٢٢٥ ر) لطلاب المستوى الثانى

(الدرجة الكلية ٨٠ درجة)

اجابة السؤال الأول (٣٥ درجة) :-

1- A relation R on Z is defined by $m R n$ if and only if $\frac{m-n}{6} \in Z$. Show that R is an equivalence relation and describe the equivalence class of [-5].

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In this case $m R n$ if and only if m − n = $6k$ for some integer k;
Firstly, R is reflexive since a - a = 6. 0,
Secondly, if $m R n$ i.e. m − n = $6k$ then $n - m$ = - $6k$ so implies $n R m$ therefore R is symmetric.
Thirdly, suppose $m R n$ and $n R s$; then there exist integers k such that m − n = $6k$ and n-s = $6k_1$. Combining these two equations gives $m - s = 6(k-k_1)$ therefore $m R s$ where (k-$k1$)is an integer. R is transitive.
Therefore $[p] = \{q \in z : q = 6k + p$, for some $k \in z\}$.

$[-5] = \{q \in z : q = 5k + -1$, for some $k \in z\}$.

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2- Show that $[\overline{(p \leftrightarrow q)}]$ logically implies $[(p \leftrightarrow \bar{q})]$

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p	q	$\bar{q}$	$p \leftrightarrow q$	$\overline{(p \leftrightarrow q)}$	$p \leftrightarrow \bar{q}$	$\overline{(p \leftrightarrow q)} \leftrightarrow p \leftrightarrow \bar{q}$
1	1	0	1	0	0	1
0	1	0	0	1	1	1
1	0	1	0	1	1	1
0	0	1	1	0	0	1

3- Let $R=\{(a,a),(a,b),(a,c),(b,b),(b,c)\}$ be a relation on the set $\{a,b,c,d\}$. What is the minimum number of elements which need to be added to R in order that it becomes:
i) reflexive; (ii) symmetric; (iii) anti-symmetric; (iv) transitive ?

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i) $R=\{(c,c),(d,d)\}$

ii) $R=\{(b,a),(c,a),(c,b)\}$

iii) $R=\phi$

iv) $R=\phi$

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4- Let A , B , C are sets, **prove that:**

I. $A\cup(B-C)=(A\cup B)-(\overline{A}\cap C)$.

II. $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

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$$A\cup(B-C)=A\cup(B\cap\overline{C})=(A\cup B)\cap(A\cup\overline{C})=(A\cup B)-\overline{(A\cup\overline{C})}$$
$$=(A\cup B)-\overline{(A\cup\overline{C})}=(A\cup B)-(\overline{A}\cap C)$$

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II. $A\times(X\cap Y)=(A\times X)\cap(A\times Y)$

Let $(a, x) \in A \times (X \cap Y)$. By the definition of the Cartesian product, this means that $a \in A$ and $x \in (X \cap Y)$. Thus $x \in X$, so (a, x) belongs to $A\times X$; and $x \in Y$, so (a, x) belongs to $A\times Y$ as well. Therefore $(a, x) \in (A\times X)\cap(A\times Y)$, which proves that $A \times (X \cap Y) \subseteq (A \times X) \cap (A \times Y)$.

To prove the subset relation the other way round as well, let $(a, x) \in (A \times X) \cap (A \times Y)$. Then $(a, x) \in (A \times X)$, so $a \in A$ and $x \in X$; and $(a, x) \in (A \times Y)$, so $a \in A$ and $x \in Y$. Therefore $a \in A$ and $x \in (X \cap Y)$ which means that the ordered pair (a, x) belongs to the Cartesian product $A \times (X \cap Y)$. Hence $(A \times X) \cap (A \times Y) \subseteq A \times (X \cap Y)$.
The conclusion that the sets $A \times (X \cap Y)$ and $(A \times X) \cap (A \times Y)$ are equal now follows, since each is a subset of the other..

أجابة السؤال الثانى (25 درجة) :-

1. Let *f*, *g* and *h* be functions $f : R \to R$ defined respectively by :

$f(x) = (5x-3), \quad g(x) = x^3, \quad h(x) = \sqrt{x^2+2}$

   i. **Find** expressions $((g \circ h) \circ f)(x)$;

   ii. **Show that** $(f \circ g)(x)$ is bijective and **find** its inverse

2. **Design** a logic network for the following so that the output is described by the following Boolean expression: $(x_1 x_2 \oplus x_1 \oplus \overline{x_2})$.

3. **Prove that** by definition, $A-(B \cap C) = (A-B) \cup (A-C)$.

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i.

$$((g \circ h) \circ f)(x) = g(h(f(x))) = g(h((5x-3)))$$
$$= (\sqrt{(5x-3)^2+2})^3$$

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Let Z(x) = $(f \circ g)(x) = f(g(x)) = 5x^3 - 3$ .ii

To show that Z is an injection we prove that, for all real numbers *x* and *y*, *z (x)* = *z (y)* implies *x* = *y*. Now *f (x)* = *f (y)* i.e. $5x^3 - 3 = 5y^3 - 3 \Rightarrow x = y$ so *f* is injective.

To show that Z is a surjection, let *y* be any element of the codomain Z . We need to find $x \in R$ such that $Z(x) = y$. Let $x = \sqrt[3]{\frac{y+3}{5}}$. Then $x \in R$ and $f(x) = [5(\sqrt[3]{\frac{y+3}{5}})^3 - 3] = y$

so Z is surjective. To find Z $^{-1}$ we simply use its definition: if y = Z (x) then x = Z $^{-1}$(y).

Now y = Z (x) $\Rightarrow y = (5x-3)^3 \Rightarrow x = \sqrt[3]{\frac{y+3}{5}}$ Therefore $x = f^{-1}(y) = \sqrt[3]{\frac{y+3}{5}}$.

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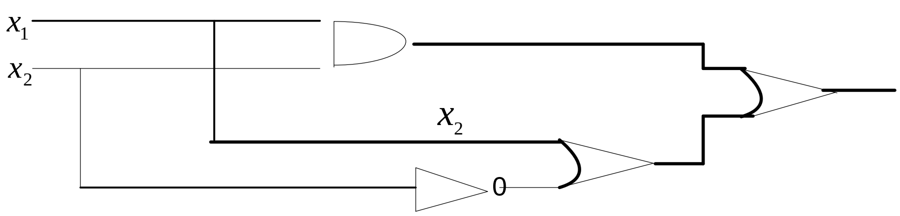
iii. **Prove that** $\mathrm{Im}(h \circ f)(x) \subseteq \mathrm{Im}\, h$

Let $c \in im(g \circ f)$. Then there exists $a \in A$ such that $(g \circ f)(a) = g(f(a)) = c$. Now let $b = f(a) \in B$; then $g(b) = c$, so $c \in im(g)$. Therefore $im(g \circ f) \subseteq im(g)$.

2. **Design** a logic network for the following so that the output is described by the following Boolean expression: $(x_1 x_2 \oplus x_1 \oplus \overline{x_2})$.



$(x_1 x_2 \oplus x_1 \oplus \overline{x_2})$.

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3. **Prove that** by definition, $A-(B\cap C)=(A-B)\cup(A-C)$.

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First we show $A-(B\cap C) \subseteq (A-B)\cup(A-C)$. Let $x \in A-(B\cap C)$. Then $x \in A$ and $x \notin B\cap C$. Hence $x \in A$ and either $x \notin B$ or $x \notin C$ (or both). Therefore either $x \in A$ and $x \notin B$ or $x \in A$ and $x \notin C$ (or both). It follows that $x \in A-B$ or $x \in A-C$ (or both).
Hence $x \in (A-B)\cup(A-C)$. We have shown that if $x \in A-(B\cap C)$ then $x \in (A-B)\cup(A-C)$. Therefore $A-(B\cap C) \subseteq (A-B)\cup(A-C)$.
Secondly we must show that $(A-B)\cup(A-C) \subseteq A-(B\cap C)$.
Let $x \in (A-B)\cup(A-C)$. Then $x \in A-B$ or $x \in A-C$ (or both) so $x \in A$ and $x \notin B$ or $x \in A$ and $x \notin C$ (or both). Hence $x \in A$ and either $x \notin B$ or $x \notin C$ (or both) which implies $x \in A$ and $x \notin B\cap C$. Therefore $x \in A-(B\cap C)$. We have shown that if $x \in (A-B)\cup(A-C)$ then $x \in A-(B\cap C)$. Therefore $(A-B)\cup(A-C) \subseteq A-(B\cap C)$. Finally, since we have shown that each set is a subset of the other, we may conclude $(A-B)\cup(A-C) = A-(B\cap C)$.

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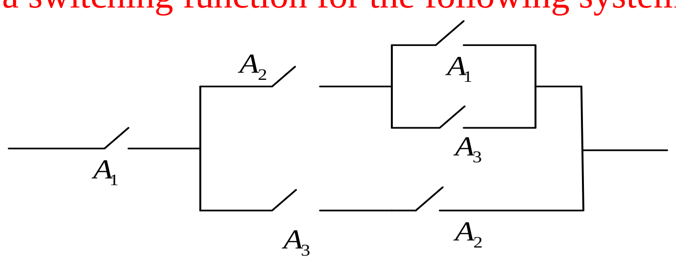
أجابة السؤال الثالث (20 درجة) :-

1. **Define** a switching function for the following system of switches:



$f(x_1, x_2, x_3) = x_1[x_2(x_1 \oplus x_3) \oplus x_3 x_2]$

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2. Describe the degree sequence of :
   1. a null graph with n vertices;
   2. The complete graph K_n;

3. An r-regular graph with n vertices
4. The complete bipartite graph $K_{n,m}$ where $n \le m$,

and **which values** of n, r, m, the graphs K_n, $K_{n,m}$ and r- regular graph are **Eulerian**?

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1. a null graph with n vertices; $(0,0,0,\ldots)$
2. The complete graph K_n; $(n-1, n-1, n-1, \ldots)$
3. An r-regular graph with n vertices $(r, r, r, \ldots)$
4. The complete bipartite graph $K_{n,m}$ where $n \le m$,
   $(n, n, n, .. n, m, m, m, .. m)$

values of n, r, m, the graphs K_n, $K_{n,m}$ and r- regular graph are **Eulerian**

K_n اذا كانت عدد فردى

$K_{n,m}$ اذا كانت n, m أعداد زوجية

r- regular graph اذا كانت r زوجية

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3. **Define** a Boolean algebra $(B, \oplus, *, \bar{\ }, 0, 1)$ and for all $b_1, b_2 \in B$, **prove that:**
$\overline{(b_1 * b_2)} = \overline{b_1} \oplus \overline{b_2}$.

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Boolean algebra consists of a set *B* together with three operations defined on that set. These are:

(a) a binary operation denoted by $\oplus$ referred to as the **sum** ;
(b) a binary operation denoted by $*$ referred to as the **product** ;
(c) an operation which acts on a single element of *B*, denoted by -,
where, for any element $b \in B$, the element $b^- \in B$ is called the **complement** of b^- (An operation which acts on a single member of a set *S* and which results in a member of *S* is called a **unary operation**.)

The following axioms apply to the set *B* together with the operations $\oplus$, $*$ and - .

B1. Distinct identity elements belonging to B exist for each of the binary operations $\oplus$ and $*$ and we denote these by **0** and **1**
respectively. Thus we have

$$b \oplus \mathbf{0} = \mathbf{0} \oplus b = b$$
$$b * \mathbf{1} = \mathbf{1} * b = b \quad \text{for all } b \in B.$$

for all $a, b, c \in B$.

$$(a * b) * c = a * (b * c)$$
$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

B2. The operations $\oplus$ and $*$ are associative, that is
B3. The operations $\oplus$ and $*$ are commutative, that is

$$a \oplus b = b \oplus a$$

$$a * b = b * a \quad \text{for all } a, b \in B.$$

B4. The operation $\oplus$ is distributive over $*$ and the operation $*$ is distributive over $\oplus$, that is

$$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$$
$$a * (b \oplus c) = (a * b) \oplus (a * c) \text{ for all } a, b, c \in B.$$

B5. For all $b \in B$, $b \oplus .b = 1$ and $b * .b = 0$.

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$(b_1 \oplus b_2) \oplus (\overline{b_1} * \overline{b_2}) = [(b_1 \oplus b_2) \oplus \overline{b_1}] * [(b_1 \oplus b_2) \oplus \overline{b_2}]$ (axiom B4)
$= [\overline{b_1} \oplus (b_1 \oplus b_2)] * [(b_1 \oplus b_2) \oplus \overline{b_2}]$ (axiom B3)
$= [(\overline{b_1} \oplus b_1) \oplus b_2] * [b_1 \oplus (b_2 \oplus \overline{b_2})]$ (axiom B2)
$= (1 \oplus b_2) * (b_1 \oplus 1)$ (axiom B5)
$= 1 * 1$ (theorem 9.4)
$= 1$ (axiom B1).

We have proved that $(b_1 \oplus b_2) \oplus \overline{b_1} * \overline{b_2}) = 1$ so that $\overline{b_1} * \overline{b_2}$ is the complement of $b_1 \oplus b_2$, i.e. $(b_1 \oplus b_2) = \overline{b_1} * \overline{b_2}$.
That $(b_1 * b_2) = \overline{b_1} \oplus \overline{b_2}$ follows from the duality principle.

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