جامعة بنـها – كلية العلوم – قسم الرياضيات

لطلاب المستوى الرابع

يوم الامتحان: الاربعاء ١٨ / ١ / ٢٠١٧ م

المادة : رياضيات متقطعة (٣١٢ ر)

الممتحن: د . / محمد السيد عبدالعال عبدالغنى

مدرس بقسم الرياضيات بكلية العلوم

اسئله + نموذج إجابه

ورقة كاملة

رياضيات متقطعة (٣١٢ ر) لطلاب المستوى الرابع

Answer the following questions: (80 marks) أجب على الاسئله التاليه (الدرجة الكلية ٨٠ درجة)

Question 1. السؤال الأول (20 درجة) :-

1- Show that $[(p \to q) \wedge (p \vee r)]$ logically implies $(q \vee r)$

2- Let $R = \{(a,a),(a,d),(b,b),(c,c),(d,e),(e,a),(d,e)\}$ be a relation on the set $\{a,b,c,d,e\}$. **What** is the minimum number of elements which need to be added to R in order that it becomes:
(i) reflexive; (ii) symmetric; (iii) anti-symmetric; (iv) transitive ?

3- Let A , B , C are sets, **prove that:**

I. $A \cap (B - C) = (A \cap B) - C$.
II. $P(A) \cap P(B) = P(A \cap B)$.

Question 2. السؤال الثانى (20 درجة)

1. If a connected planar graph G has V vertices and E edges and dividing the plane into F faces, then **prove that**: $F = E - V + 2$.

2. A relation R on $Z^+ \times Z^+$ is defined by $(m,n)\,R\,(p,q)$ if and only if $mq = np$. Show that R is an equivalence relation and describe the equivalence class of (2,5).

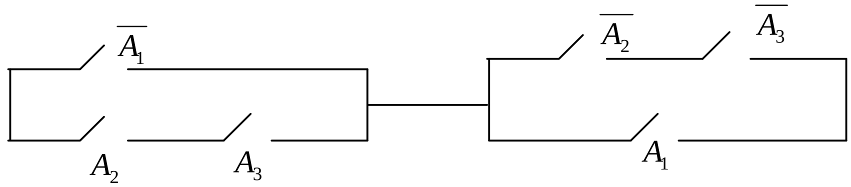
3. **Design** a logic network for the following so that the output is described by the following Boolean expression: $(x_1 \oplus \overline{x_2} \oplus x_3)\,\overline{x_1}$.

انظر خلف الورقة

Question 3.

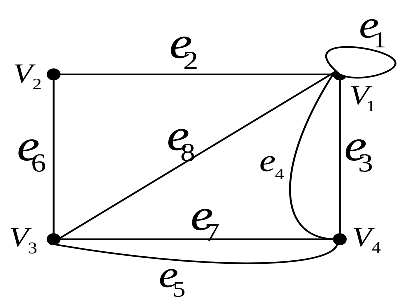
السؤال الثالث (20 درجة)

1. **Define** a switching function for the following system of switches:



2. **Define** a Boolean algebra $(B, \oplus, *, ^{-}, 0, 1)$ and for all $b_1, b_2 \in B$, **prove that**:
There is only one element $\overline{b_1} \in B$ such that $b_1 \oplus \overline{b_1} = 1$ and $b_1 * \overline{b_1} = 0$.

3. **Find** the matrix A^2, where A be the adjacency matrix, for the following graph:
and **write** all edge sequences of length 2 joining v_1, v_4.



Question 4.

السؤال الرابع (20 درجة)

1. For any propositions *p*, *q*, *r*, **Prove that:** $(p \leftrightarrow q) \leftrightarrow r \equiv p \leftrightarrow (q \leftrightarrow r)$.

2. **Define** *Trees, the complete graph* K_n, *the complete bipartite graph* $K_{r,s}$ and, for **which values** of n, r, s, the graphs K_n, $K_{r,s}$ are **Eulerian**?

3. Let *f*, *g* and *h* be functions $f : R \to R$ defined respectively by :
$f(x) = (5x^2 + 3), \quad g(x) = x^3, \quad h(x) = \sqrt{x^2 + 2}$
Find expressions $(f \circ h)(x), \ (h \circ g)(x)$;

انتهت أسئلة

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مع أطيب تمنياتى بالتوفيق والنجاح

Good Luck !

د. محمد السيد عبدالعال

نموذج اجابه لأمتحان رياضيات متقطعة (٢٢٥ ر) لطلاب المستوى الثانى

(الدرجة الكلية ٨٠ درجة)

Question 1. السؤال الأول (20 درجة) :-

1- Show that $[(p \to q) \wedge (p \vee r)]$ logically implies $(q \vee r)$

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p	q	r	$p \to q$	$p \vee r$	$(p \to q) \wedge (p \vee r)$	$q \vee r$	$[(p \to q) \wedge (p \vee r)] \to (q \vee r)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	0	1	0	1	0	1	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	0	1	1	1	1	1	1
0	0	0	1	0	0	0	1

2- Let $R = \{(a,a),(a,d),(b,b),(c,c),(d,e),(e,a),(d,c)\}$ be a relation on the set $\{a,b,c,d,e\}$. **What** is the minimum number of elements which need to be added to R in order that it becomes:
(ii) reflexive; (ii) symmetric; (iii) anti-symmetric; (iv) transitive ?

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i) $R = \{(d,d),(e,e)\}$

ii) $R = \{(d,a),(e,d),(c,d),(a,e)\}$

iii) $R = \phi$

iv) $R = \{(a,e),(d,a),(e,d),(a,c)\}$

4- Let A, B, C are sets, **prove that:**

III. $A \cap (B - C) = (A \cap B) - C$.

IV. $P(A) \cap P(B) = P(A \cap B)$.

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$$A \cap (B - C) = A \cap (B \cap \overline{C}) = (A \cap B) \cap \overline{C}) = (A \cap B) - C$$

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II. $P(A) \cap P(B) = P(A \cap B)$

Let $X \in P(A) \cap P(B) \Leftrightarrow X \in P(A) \wedge X \in P(B)$

$\Leftrightarrow X \subseteq A \wedge X \subseteq B \Rightarrow X \subseteq A \cap B \Leftrightarrow X \in P(A \cap B)$

$\Leftrightarrow P(A) \cap P(B) = P(A \cap B)$

Question 2. السؤال الثانى (20 درجة)

1. If a connected planar graph G has V vertices and E edges and dividing the plane into F faces, then **prove that**: $F = E - V + 2$.

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The proof is by induction on the number of edges of G. If $E = 0$ then $V = 1$ (G is connected, so there cannot be two or more vertices) and there is a single face (consisting of the whole plane except the single vertex), so $F= 1$. therefore holds in this case. Suppose, now, that the theorem holds for all graphs with fewer than n edges. Let G be a connected planar graph with n edges; that is $|E| = n$. If G is a tree, then $|V| = n + 1$ and $|F| = 1$, so the theorem holds in this case too. If G is not a tree choose any cycle in G and remove one of its edges. The resulting graph G is connected, planar and has $n - 1$ edges, $|V|$ vertices and $|F| - 1$ faces. By the inductive hypothesis, Euler's formula holds for G_: $|F| - 1 = (|E| - 1) - |V| + 2$
so $|F| = |E| - |V| + 2$ as required

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1. A relation R on $Z^+ \times Z^+$ is defined by $(m, n) R (p, q)$ if and only if $mq = np$. Show that R is an equivalence relation and describe the equivalence class of (2,5).

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For all positive integers *a* and *b*, *a b* = *b a*, so *(a, b) R (a, b)* for every *(a, b)* ∈ *A*, Therefore *R* is reflexive.
R is symmetric since if (*a, b) R (c, d*) then *a d* = *b c* which implies that *c b* = *d a*, so (*c, d) R (a, b*).
To show that *R* is transitive, suppose *(a, b) R (c, d*) and (*c, d) R (e, f*). This means that *a d* = *b c* and *c f* = *d e*. we have *a f* = *b e*.
This means that *(a, b) R (e, f*). Therefore *R* is transitive
And *R* is an equivalence relation

$$[(2,5)] = \{(x,y) \in Z^+ \times Z^+ : 2y=5x \}.$$

2. Design a logic network for the following so that the output is described by the following Boolean expression $(x_1 \oplus \overline{x_2} \oplus x_3)\overline{x_1}$.

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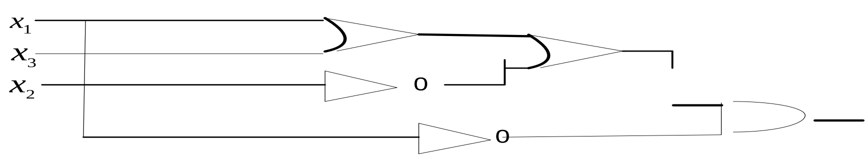
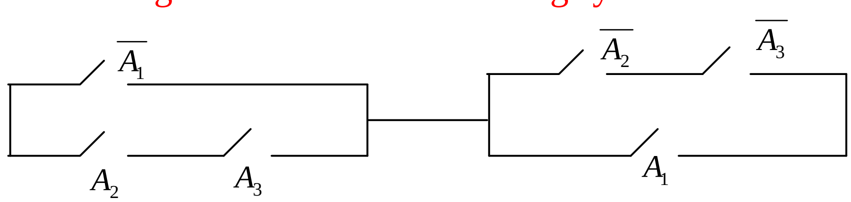


Question 3. السؤال الثالث (20 درجة)

1. **Define** a switching function for the following system of switches:



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$$f(x_1, x_2, x_3) = (\overline{x_1} \oplus x_2 x_3)(\overline{x_2}\,\overline{x_3} \oplus x_1)$$

2. **Define** a Boolean algebra $(B, \oplus, *, ^{-}, 0, 1)$ and for all $b_1, b_2 \in B$, **prove that:**
There is only one element $\overline{b_1} \in B$ such that $b_1 \oplus \overline{b_1} = 1$ and $b_1 * \overline{b_1} = 0$.

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Boolean algebra consists of a set *B* together with three operations defined on that set. These are:

(a) a binary operation denoted by ⊕ referred to as the **sum** ;

(b) a binary operation denoted by ∗ referred to as the **product** ;

(c) an operation which acts on a single element of *B*, denoted by -, where, for any element *b* ∈ *B*, the element b^- ∈ *B* is called the **complement** of b^- (An operation which acts on a single member of a set *S* and which results in a member of *S* is called a **unary operation**.)

The following axioms apply to the set *B* together with the operations ⊕, ∗ and - .

B1. Distinct identity elements belonging to *B* exist for each of the binary operations ⊕ and ∗ and we denote these by **0** and **1** respectively. Thus we have

$$b \oplus \mathbf{0} = \mathbf{0} \oplus b = b$$
$$b * \mathbf{1} = \mathbf{1} * b = b \quad \text{for all } b \in B.$$

for all *a*, *b*, *c* ∈ *B*.

$$(a * b) * c = a * (b * c)$$
$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

B2. The operations ⊕ and ∗ are associative, that is

B3. The operations ⊕ and ∗ are commutative, that is

$$a \oplus b = b \oplus a$$

$$a * b = b * a \quad \text{for all } a, b \in B.$$

B4. The operation ⊕ is distributive over ∗ and the operation ∗ is distributive over ⊕, that is

$$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$$
$$a * (b \oplus c) = (a * b) \oplus (a * c) \text{ for all } a, b, c \in B.$$

B5. For all b ∈ B, b ⊕ .b = 1 and b ∗ .b = 0.

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Suppose that .b1 and .b2 are both complements of an element b of a Boolean algebra (P(S),∪,∩, -, φ., S) = (B,⊕, ∗, -, 0, 1). This means that

$$b \oplus .b_1 = .b_1 \oplus b = 1 , \qquad b \oplus .b_2 = .b_2 \oplus b = 1$$
$$b * .b_1 = .b_1 * b = 0 , \quad b * .b_2 = .b_2 * b = 0., \quad .b_{1=}\overline{b_1}$$

Thus we have

.b1 = .b1 ∗ 1 (axiom B1)

= .b1 ∗ (b ⊕ .b2)

= (.b1 ∗ b) ⊕ (.b1 ∗ .b2) (axiom B4)

= 0 ⊕ (.b1 ∗ .b2)

= 0 ⊕ (.b2 ∗ .b1) (axiom B3)

= (.b2 ∗ b) ⊕ (.b2 ∗ .b1)

= .b2 ∗ (b ⊕ .b1) (axiom B4)
= .b2 ∗ 1
= .b2 (axiom B1).

We have shown that .b1 = .b2 and so we can conclude that the complement is unique.

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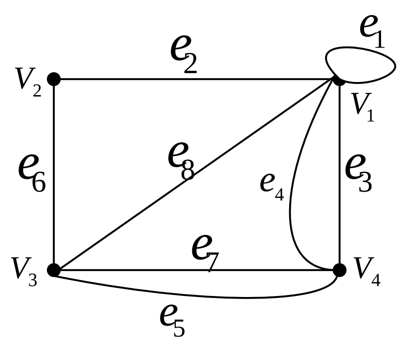
3. **Find** the matrix A^3, where A be the adjacency matrix, for the following graph: and **write** all edge sequences of length 3 joining v_2, v_3.



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$$A=\begin{pmatrix}1&1&1&2\\1&0&1&0\\1&1&0&2\\2&0&2&0\end{pmatrix},\quad A^3=\begin{pmatrix}7&2&6&\underline{\underline{4}}\\2&1&1&4\\6&1&6&2\\4&4&2&8\end{pmatrix}$$

$e_1\, e_3$; $e_1\, e_4$; $e_8\, e_7$; $e_8\, e_5$.

Question 4. السؤال الرابع (20 درجة)

1. For any propositions *p*, *q*, *r*, **Prove that:** $(p\leftrightarrow q)\leftrightarrow r \equiv p\leftrightarrow(q\leftrightarrow r)$.

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p	q	r	$(p\leftrightarrow q)$	$(p\leftrightarrow q)\leftrightarrow r$	$(q\leftrightarrow r)$	$p\leftrightarrow(q\leftrightarrow r)$
1	1	1	1	1	1	1
1	1	0	1	0	0	0
1	0	1	0	0	0	0
1	0	0	0	1	1	1
0	1	1	0	0	1	0

0	1	0	0	1	0	1
0	0	1	1	1	0	1
0	0	0	1	0	1	0

$$\therefore (p \leftrightarrow q) \leftrightarrow r \equiv p \leftrightarrow (q \leftrightarrow r)$$

2. **Define** *Trees, the complete graph* K_n , *the complete bipartite graph* $K_{r,s}$ and, for **which values** of *n, r, s* , the graphs K_n , $K_{r,s}$ are **Eulerian**?

الحــــــــــــــــل

A trees is a connected graph which contains no cycles.

the complete graph K_n is a simple graph in which every pair of distinct vertices is joined by an edge.

A complete bipartite graph is a bipartite graph such that every vertex of V_1 is joined to every vertex of V_2 by a unique edge.
An Eulerian path in a graph G is a closed path which includes every edge of G . A graph is said to be Eulerian if it has at least one Eulerian path.
The complete graph K_n is (n−1)-regular–every vertex has degree n−1. Since it is connected, K_n is Eulerian if and only if n is odd (so that n − 1is even).
A complete bipartite graph $K_{r,s}$ is Eulerian if and only if *r,s* is even.

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3. Let *f, g* and *h* be functions $f : R \to R$ defined respectively by :

$$f(x) = (5x^2 + 3), \quad g(x) = x^3, \quad h(x) = \sqrt{x^2 + 2}$$

Find expressions $(f \circ h)(x), \ (h \circ g)(x)$;

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$$(f \circ h)(x) = f(h(x)) = f(\sqrt{x^2 + 2}) = 5(x^2 + 2) + 3$$

$$(h \circ g)(x) = h(g(x)) = h(x^3) = \sqrt{(x^3 + 2)}$$

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