كليــة : العلــوم　　　　　　　　　　　　المستوى : الرابع

قسم : الرياضيــات　　　　　　　　　　　　الماده : توبو لوجى و هندسه

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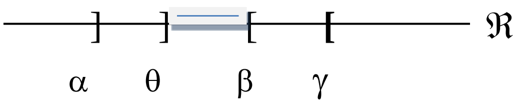
الإجابه النموزجيه :

(1)

(T1) Trivially : $\Re =]-\infty, \infty[\in \mathcal{U}$ and $\phi =]\alpha, \alpha[\in \mathcal{U}$

(T2) Let $G, H \in \mathcal{U}$. Then $G =]\alpha, \beta[$, $H =]\theta, \gamma[$, so that

$$G \cap H =]\alpha, \beta[\cap]\theta, \gamma[= \begin{cases} \phi \in \mathcal{U}, & \text{if } \beta \le \theta \\ ]\theta, \beta[\in \mathcal{U}, & \text{if } \alpha < \theta < \beta < \gamma \\ ]\alpha, \beta[\in \mathcal{U}, & \text{if } \theta < \alpha < \beta < \gamma \\ ]\alpha, \gamma[\in \mathcal{U}, & \text{if } \theta < \alpha < \gamma < \beta \end{cases}$$



In each case, we have, $G \cap H$ belongs to $\mathcal{U}$.

(T3) The union of an arbitrary number of an open intervals is again an open interval.

this topology is called usual topology on $\Re$. (١)

(T1) Obviously $\phi \in \mathcal{P}$ and since $p \in X$, so $X \in \mathcal{P}$.

(T2) Let $G, H \in \mathcal{P}$. Then $p \in G$ and $p \in H$,

therefore, $p \in G \cap H$.

Hence, $G \cap H \in \mathcal{P}$.

(T3) Let $H_i \in \mathcal{P}$, where $i \in I$. Then $p \in H_i$, $\forall$ $i \in I$,

therefore, $p \in \bigcup_{i \in I} H_i$.

Hence, $\bigcup_{i \in I} H_i \in$ ₽.

That is ₽ is a topology on X.

Solution. Since

$$₽ \cup ξ \subseteq P(X) = Đ \qquad \ldots\ldots(1)$$

On other hand, by taking . $H \in P(X) = Đ$, we get two probabilities

(i) $p \in H \;\Rightarrow\; H \in ₽ \;\Rightarrow\; H \in ₽ \cup ξ$

(ii) $p \notin H \;\Rightarrow\; H \in ξ \;\Rightarrow\; H \in ₽ \cup ξ$.

Therefore

$$Đ = P(X) \subseteq ₽ \cup ξ \qquad \ldots\ldots (2)$$

From (1), (2) we have, $₽ \cup ξ = Đ$.

(2)

From definitions we have

$$\mathit{int}\, A \cap \mathit{ext}\, A = \mathit{int}\, A \cap (\mathit{int}\, A)^c = \phi;$$

$$\mathit{int}\, A \cap b(A) = \mathit{int}\, A \cap [(\mathit{int}\, A)^c \cap (\mathit{ext}\, A)^c]$$

$$= [\mathit{int}\, A \cap (\mathit{int}\, A)^c] \cap (\mathit{ext}\, A)^c$$

$$= \phi \cap (\mathit{ext}\, A)^c = \phi \quad \text{and}$$

$$\mathit{ext}\, A \cap b(A) = \mathit{ext}\, A \cap [(\mathit{int}\, A)^c \cap (\mathit{ext}\, A)^c]$$

$$= [\mathit{ext}\, A \cap (\mathit{ext}\, A)^c] \cap (\mathit{int}\, A)^c$$

$$= \phi \cap (\mathit{int}\, A)^c = \phi. \quad \text{Also,}$$

$$\mathit{int}\, A \cup \mathit{ext}\, A \cup b(A) = \mathit{int}\, A \cup \mathit{ext}\, A \cup [(\mathit{int}\, A)^c \cap (\mathit{ext}\, A)^c]$$

$$= [\mathit{int}\, A \cup \mathit{ext}\, A \cup (\mathit{int}\, A)^c] \cap [\mathit{int}\, A \cup \mathit{ext}\, A \cup (\mathit{ext}\, A)^c]$$

$= [int\, A \cup (int\, A)^c \cup ext\, A] \cap [int\, A \cup ext\, A \cup (ext\, A)^c]$

$= [\, X \cup ext\, A] \cap [\, int\, A \cup X]$

$= X \cap X$

$= X.$

It follows from part (i) that $A' \subseteq (A \cup B)'$ and $B' \subseteq (A \cup B)'$. Then

$A' \cup B' \subseteq (A \cup B)'$, (1).

Conversely, we must prove that $(A \cup B)' \subseteq A' \cup B'$.

Take $\alpha \notin A' \cup B'$, we have $\alpha \notin A'$ and $\alpha \notin B'$ and hence there exist neighborhoods $V, W \in \mathcal{N}_\alpha$ such that

$(V - \{\alpha\}) \cap A = \phi$ and $(W - \{\alpha\}) \cap B = \phi$.

But $V \cap W \in \mathcal{N}_\alpha$ satisfies $[(V \cap W) - \{\alpha\}] \cap (A \cup B) = \phi$.

Then $\alpha \notin (A \cup B)'$, which proves that

$(A \cup B)' \subseteq A' \cup B'$, (2).

From (1), (2) we get $(A \cup B)' = A' \cup B'$.

If $A \subseteq C$, $B \subseteq C$, then we have $A \cap B \subseteq C$

$\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$.

(3)

In the indiscrete space $(X, \mathfrak{J})$ and every $\alpha \in X$ we have $\mathcal{N}_\alpha = \{X\}$.

If A is a singleton say $A = \{\gamma\}$ we get

$(X - \{\gamma\}) \cap A = (X - \{\gamma\}) \cap \{\gamma\} = \phi$,

that is $\gamma \notin A'$.

For any point $\theta \neq \gamma$, we get

$(X - \{\theta\}) \cap A = (X - \{\theta\}) \cap \{\gamma\} = \{\gamma\} \neq \phi$.

Hence, $A' = X - \{\gamma\}$.

But if A is a subset containing more than one point, we get $A' = X$.

(i) implies (ii) : Let (i) be holds, that is $h\ (X, \tau) \longrightarrow (Y, \sigma)$ is continuous and F is σ-closed set, then F^c is σ-open.

Hence, by (i) $h^{-1}(F^c)$ is τ-open. So

$h^{-1}(F^c) = h^{-1}[Y - F] = h^{-1}(Y) - h^{-1}(F) = X - h^{-1}(F)$, which is τ-open.

Thus $h^{-1}(F)$ is τ-closed set.

(ii) implies (i) : Let (ii) be holds and H be σ-open set, then H^c is σ- closed.

Hence, by (ii), we get $h^{-1}(H^c)$ is τ-closed set. So

$h^{-1}(H^c) = h^{-1}[Y - H] = h^{-1}(Y) - h^{-1}(H) = X - h^{-1}(H)$, which is τ-closed.

That is $h^{-1}(H)$ is τ-open set, therefore h is continuous.

(ii) implies (iii) : Let (ii) be holds and $A \subseteq X$. Since $h(A) \subseteq \overline{h(A)}$, then

$A \subseteq h^{-1}[h(A)]$, by Theorem 1.1

$\subseteq h^{-1}[\overline{h(A)}]$, clear

But, $\overline{h(A)}$ is σ-closed, so by (ii) we have $h^{-1}[\overline{h(A)}]$ is τ-closed.

Thus, $\overline{A} \subseteq h^{-1}[\overline{h(A)}]$ and hence by Theorem 1.1 again, we have

$h(\overline{A}) \subseteq h\{h^{-1}[\overline{h(A)}]\} \subseteq \overline{h(A)}$.

(iii) implies (ii) : Let (iii) be holds, that is $h(\overline{A}) \subseteq \overline{h(A)}$, for every $A \subseteq X$.

Take F is σ-closed set and put $B = h^{-1}(F)$. Then by (iii) we have

$\overline{B} \subseteq h^{-1}[h(\overline{B})]$, by Theorem 1.1

$= h^{-1}[h(\overline{h^{-1}(F)})]$

$\subseteq h^{-1}[\overline{h(h^{-1}(F))}]$, by hypothis

$\subseteq h^{-1}\overline{(F)}$, by Theorem 1.1

$= h^{-1}(F)$, for F is σ-closed

$= B$

$\subseteq \overline{B}$.

This meaning that $\overline{B} = h^{-1}(F)$ which is τ-closed set.

(5)

. If $\mathcal{B}$ is a base of some topology τ ($\mathcal{B} \subset \tau$), H and M are members of $\mathcal{B}$ and $\alpha \in H \cap M$, we have $H, M \in \tau$. Thus $H \cap M \in \tau$, therefore $H \cap M$ is a union of some members in $\mathcal{B}$. So there is a member W in $\mathcal{B}$ which α belongs to it and which is a subset of $H \cap M$.

Conversely, let $\mathcal{B}$ be a family with the specified property and τ be the family of all unions of members of $\mathcal{B}$, we have :

(T1) The empty set ϕ is the void family of members of $\mathcal{B}$.

(T2) Let G_1 and G_2 be two members of τ. If $\alpha \in G_1 \cap G_2$ $(G_1 \cap G_2 \neq \phi)$, then at this, we may choose H_1 and H_2 in $\mathcal{B}$ such that $\alpha \in H_1 \subseteq G_1$ and $\alpha \in H_2 \subseteq G_2$. So there is a member W in $\mathcal{B}$ satisfies

$\alpha \in W \subseteq H_1 \cap H_2 \subseteq G_1 \cap G_2$.

Consequently $G_1 \cap G_2$ is the union of some members of $\mathcal{B}$.

Therefore $G_1 \cap G_2$ is a member of τ.

(T3) A union of members of τ is itself a union of members of $\mathcal{B}$ and is therefore a member of τ.

Hence τ is a topology on X.