1. (b) Using the dimensional analysis, derive an expression for the time period of oscillation of a simple pendulum. Assuming that the time period depends on (i) mass, (ii) length and (iii) acceleration due to gravity.

---------------------------- Solution -------------------------------------

Assume that t, m, ℓ and g are related through the equation:

$$t \propto m^x \ell^y g^z$$

$$t = k\, m^x \ell^y g^z$$

By using the dimensional method

$$T = M^x L^y (L T^{-2})^z$$

$$M^0 L^0 T^1 = M^x L^{y+z} T^{-2z}$$

Comparison the powers of M, L and T on both sides

$$x = 0, \quad y + z = 0, \quad -2z = 1$$

Solving the three equations,

$$x = 0, \quad y = \tfrac{1}{2}, \quad z = -\tfrac{1}{2}$$

$$\therefore\ t = k\sqrt{\frac{\ell}{g}}$$

2. (a) Define Young's modulus

---------------------------- Solution -------------------------------------

Young's modulus of elasticity (Y): It is defined as the ratio of normal stress to longitudinal strain.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F_\perp / A}{\Delta\ell / \ell} = \frac{F_\perp\, \ell}{A\, \Delta\ell}$$

shearing modulus

Shear modulus (S): It is defined as the ratio of tangential stress to shearing strain.

$$S = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F_{||}/A}{\theta} = \frac{F_{||}}{A\,\theta}$$

Poisson's ratio

Poisson's ratio is defined as the ratio of secondary strain per unit stress to the longitudinal strain per unit stress.

$$\sigma = \frac{-\,dr/r}{d\ell/\ell}$$

The Poisson's ratio σ has no units as it is a ratio of two numbers. For most of substances, the value of σ is $\frac{1}{2}$.

2. (b) For a wire under tension, write on the steps that it follow

--------------------------- Solution -----------------------------------

We may now consider the relation between each of the three kinds of stress and its corresponding strain. When any stress is plotted against the appropriate strain, the resulting stress strain diagram is found to have a shape as given in Fig. (5).

- **Elastic region (OP):** During the first portion of the curve (OP), the stress and strain are proportional until the point P, the proportional limit, is reached. The fact that there is a region in which stress and strain are proportional is called ***Hook's law***. If the load is removed at any point between O and P, the material be return to its original length. In the region OP, the material is said to be elastic and the point P is called the ***elastic limit***. Up to this point, the force exerted by the material are conservative; when the material returns to its original shape, work done in producing the deformation is recovered.

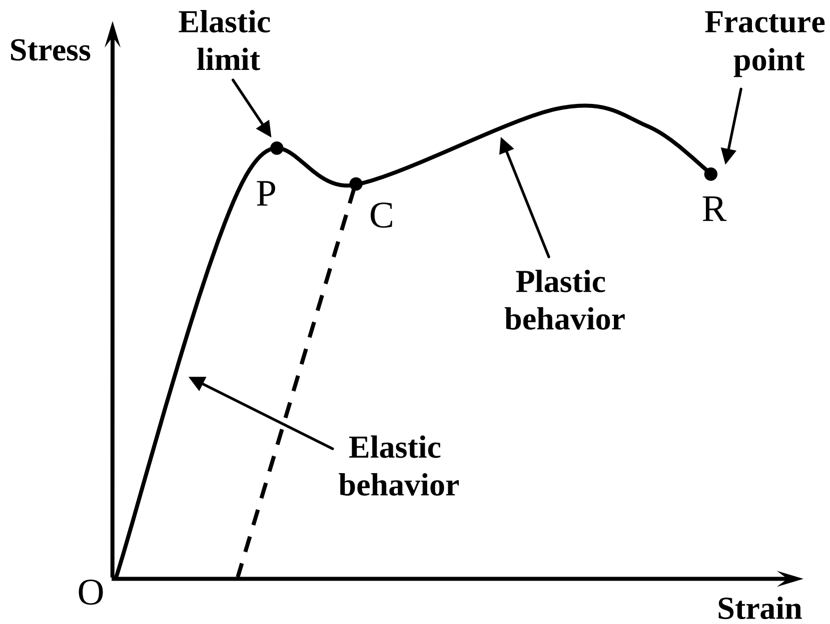

Figure (5)

- **Plastic region PR:** If the material is loaded, the strain increases rapidly, and when the load is removed at some point, say C, the material does not come back to its original length but traverses the dashed line in Fig. (5). From P to R, the material is said to undergo plastic deformation. Increase of load beyond C produces a large increase in strain until a point R is reached at which fracture take place.

3. (a) An object of mass m is hung from a spring and set into oscillation. Prove that the period of oscillation is given by $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

---Solution----------------------------------

As a model for simple harmonic motion, consider a block of mass m attached to the end of a spring, with the block free to move on a horizontal, frictionless surface, as in Fig. (4). When is neither stretched, the block is at the position called the equilibrium position of the system, which identify as $x = 0$.

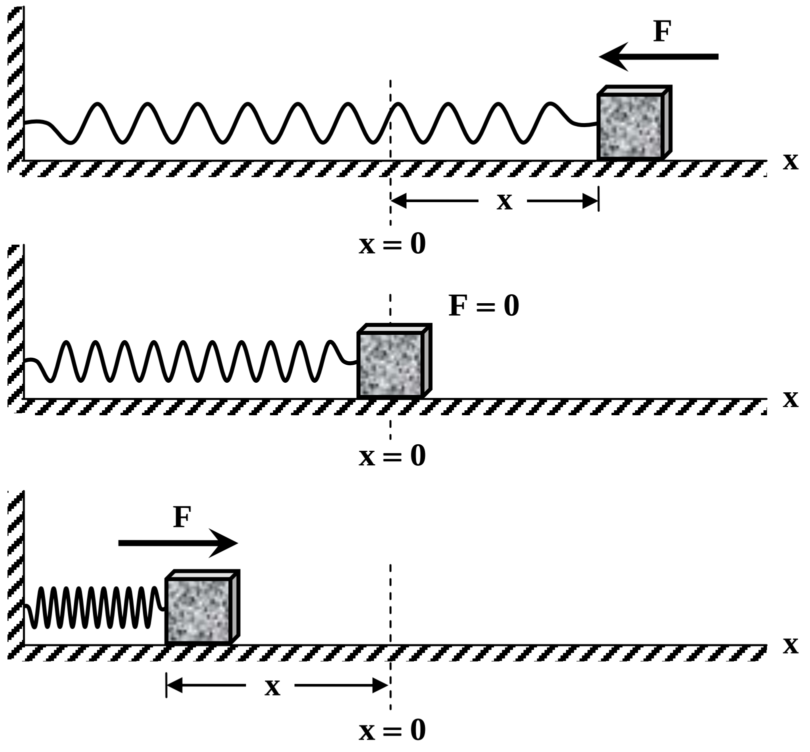

Figure (4)

We can understand the motion in Fig. (4) by recalling that when the block is displaced to the position x, the spring exerts on the block a force that is proportional to the position and given by Hook's law:

$$F = -k x$$

We call this a ***restoring force*** because it is a always directed toward the equilibrium position and therefore *opposite* the displacement from equilibrium. That is, when the block is displaced to the right of $x = 0$ in Fig. (4a), then the position is positive and the restoring force is directed to the left. When the block is displaced to the left of $x = 0$, then the position is negative and the restoring force is directed to the right.

Applying Newton's second law to the motion of the block, we obtain:

$$F = -k x$$

$$m\ddot{x} = -k x$$

$$\ddot{x} = -\frac{k}{m} x$$

This equation is similar to the equation of simple harmonic motion

$$\ddot{y} = -\omega^2 y$$

Comparing the last two equations:

$$\omega = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

3. (b) A block on the end of a spring is pulled to position $x = A$ and released. In one full cycle of its motion, through what is the total distance does it travel?.

---Solution-----------------------------------

One full cycle of its motion, the total distance it travel is

4A

4 Describe the principle, construction and working of a thermo electric thermometer

--Solution-----------------------------------

The principle underlying a thermoelectric thermometer is that when one junction of two different metals such as iron and copper is heated keeping the other cold an emf is generated and a current flows through the circuit, see Fig. (6). This is known as ***Seebeck effect***. The magnitude of the emf generated is proportional to the temperature of the hot junction if that of the cold junction is kept constant. Variation of thermo emf with temperature is given from the expression:

$$E = \alpha T + \beta T^2$$

where T is the temperature of the hot junction, α and β are constants. It has been found that for temperature up to $300^{o}C$, copper constantan and iron constantan are good as they give thermo emf of the order of 40 to 60 microvolt per degree temperature difference between junctions.

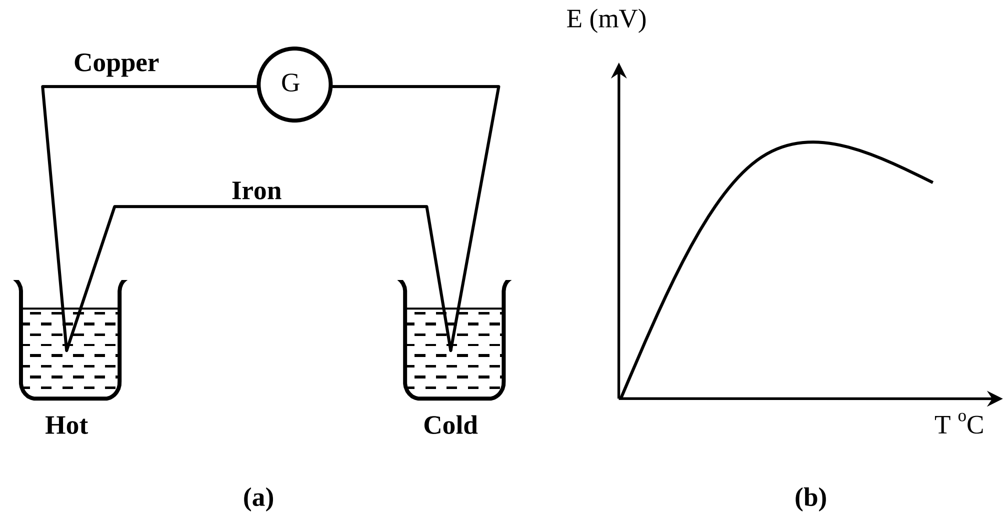

Fig. (6): (a) Thermocouple thermometer. (b) Thermocouple characteristic

5. Describe a method to determine the specific heat of a solid by the method of mixtures

---Solution-----------------------------------

Mixture method is the one most commonly used in the laboratory for determining the specific heat of solids. The apparatus consists of two parts, the heater H and the calorimeter C, see Fig. (3). The heater consists of two coaxial cylinders, the annular space between them being supplied with a steady flow of steam. The top of the air chamber (inner one) is closed with a cork while the bottom by a trap door through which the solid may be dropped into the calorimeter. The calorimeter is a copper vessel placed in a wooden box packed with wood to reduce loss of heat by conduction. The calorimeter is provided with a copper stirrer and a sensitive thermometer.

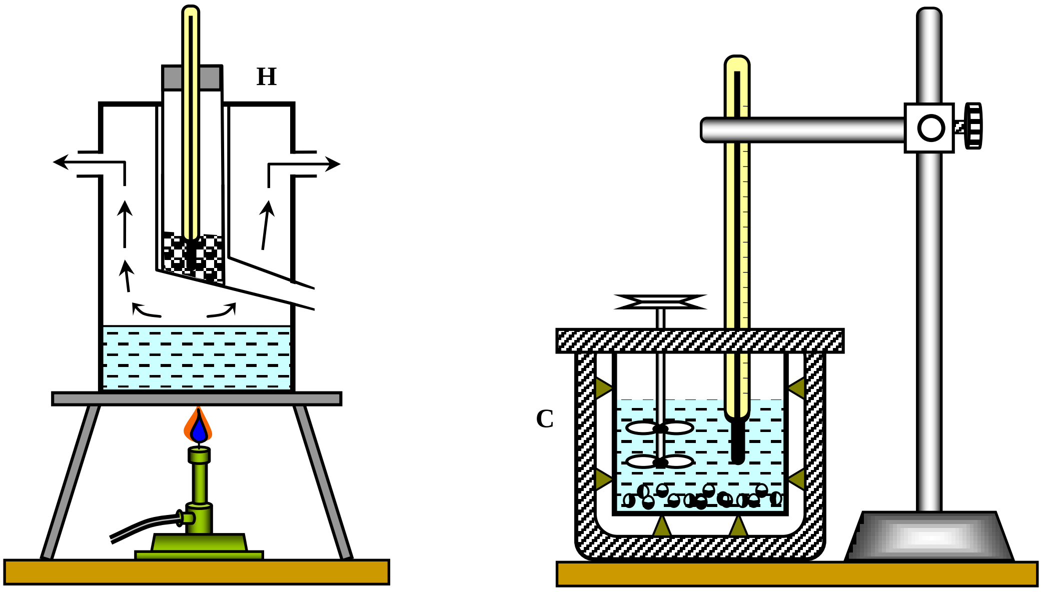

Fig. (3): Determination of specific heat of a solid by mixture method

The solid, in the form of a small pieces, is weighted and suspended inside the heater by a thread passing through the cork. Steam is passed through the heater from a boiler, so that the solid may be heated without actual contact with steam or water. While the solid is heated, the empty dry calorimeter, with the stirrer is weighted. Water is taken in the calorimeter and the calorimeter and contents are weighted again. The mass of water taken is then readily found. The calorimeter is placed back inside the wooden box and the temperature of the water is noted.

When the solid has attained the steady maximum temperature, the calorimeter is pushed under the trap door of the heater and the solid dropped into the calorimeter. The contents of the calorimeter are well stirred and the highest temperature reached is noted.

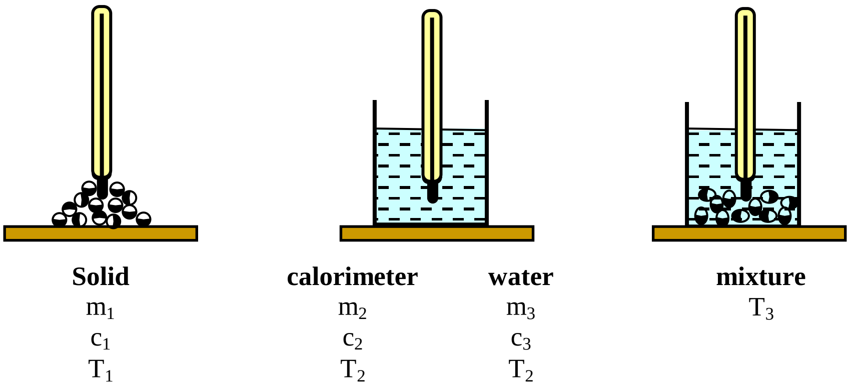

Fig. (4): Data taken in the mixture method

Suppose c_1 is the specific heat of a given solid. m_1 grams of the solid at $T_1\,^oC$ is added to m_3 of water at $T_2\,^oC$ in a calorimeter whose mass is m_2 and its specific heat is c_2. Let $T_3\,^oC$ be the final temperature of the mixture, see Fig. (4). By equating the heat lost by the solid Q_s to the heat gained by the calorimeter and water $(Q_c + Q_w)$ we get

$$\text{Heat lost by solid} = \text{Heat gained by (calorimeter + water)}$$

$$Q_s = Q_c + Q_w$$

$$\boxed{m_1 c_1 (T_3 - T_1) = (m_2 c_2 + m_3 c_3)(T_3 - T_2)} \qquad (8)$$

from which c_1 can be determined.