كلية العلوم   نموذج إجابة مادة: الفيزياء الاحصائية

الفرقة الثالثة علوم (فيزياء) (تخلفات)   د./ صلاح عيد ابراهيم حمزة

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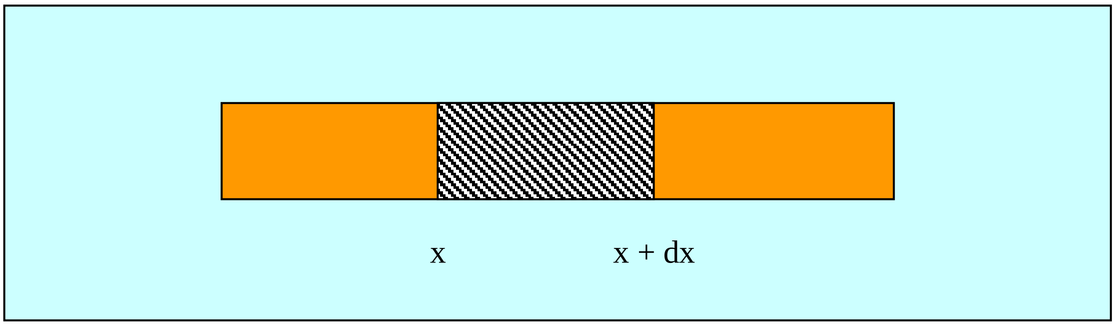
Q1. (a) Write a short note about the vibrational spectrum of crystals.

-------------------- Solution ----------------------------

Let us examine the propagation of an elastic wave in a long bar. The wave equation in one dimension is



$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = 0 \qquad (1)$$

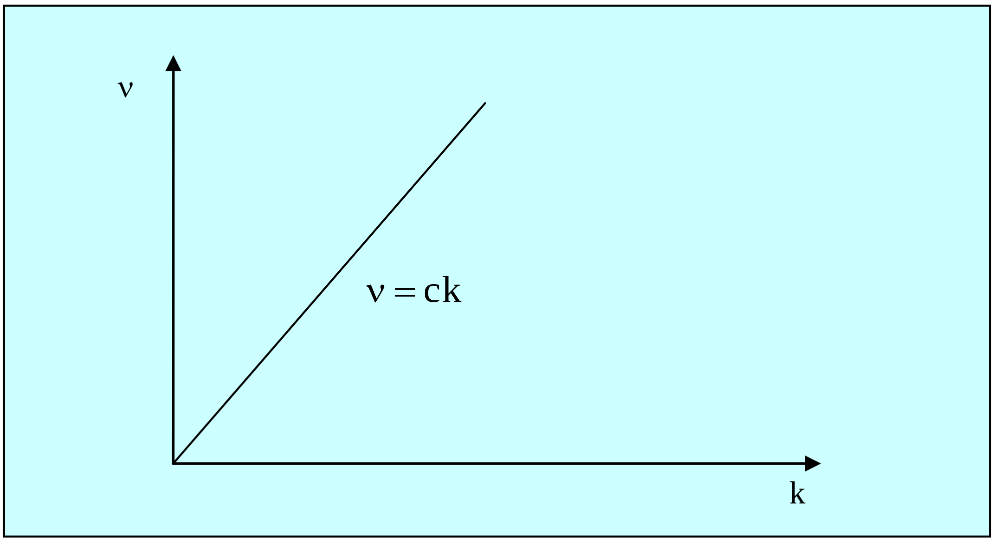
The solution of this equation is

$$\varphi = Ae^{i(kx - \nu t)} \qquad (2)$$

Substituting Eq. (2) in (1) leads to

$$\nu = ck \qquad (3)$$

The last equation is known as the dispersion relation which represents a straight line as in the figure

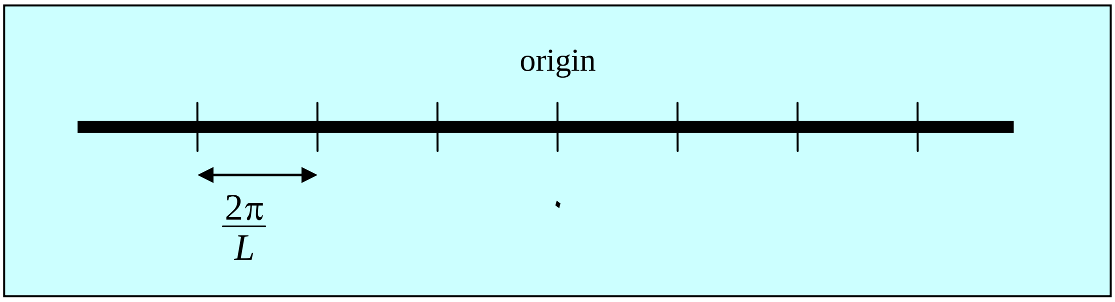


The boundary conditions require that

$$\varphi(0) = \varphi(L) \qquad (4)$$

Substituting by Eq. (2) in (4) gives

$$k = n\frac{2\pi}{L}, \qquad n = 0, \pm 1, \pm 2, \ldots \qquad (5)$$



The density of states is

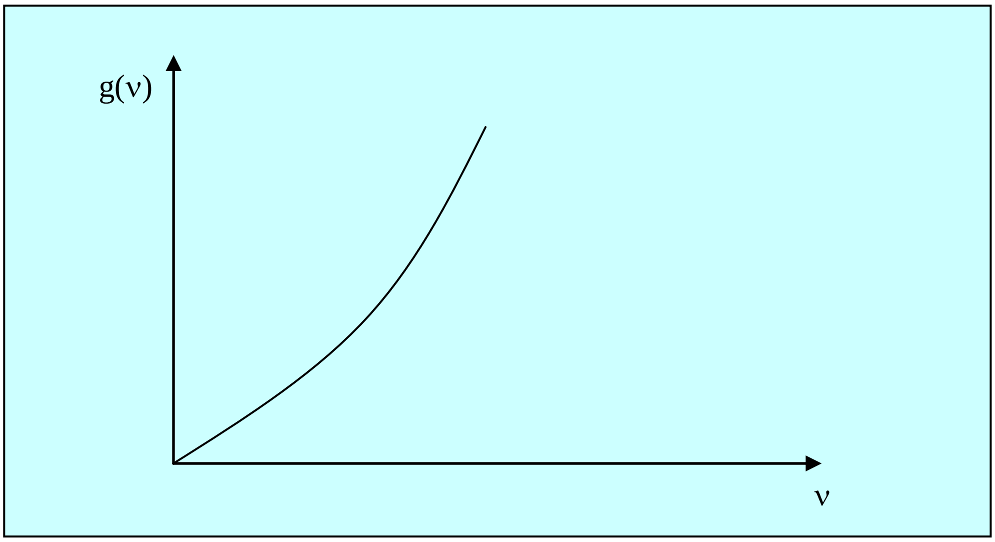
$$g(\nu)d\nu = \frac{L}{2\pi}dk \qquad (6)$$

In one dimension

$$g(\nu) = \frac{L}{2\pi}\frac{1}{c}$$

In three dimension

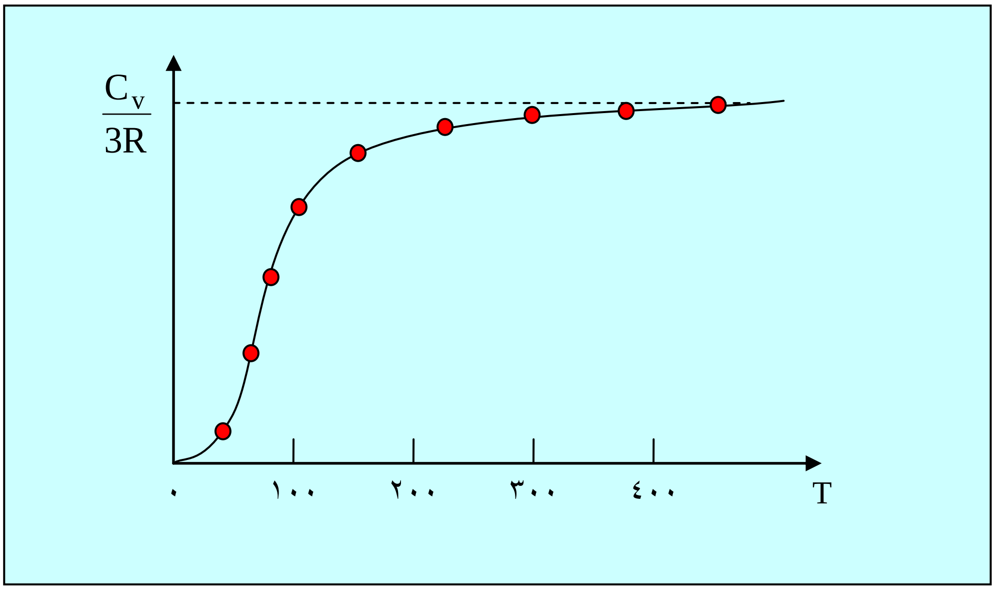
$$g(\nu) = \frac{3V}{2\pi^2}\frac{\nu^2}{c^3} \qquad (7)$$



Q1. (b) Discus the classical theory interpretation for Duliong-Petit law of specific heat.

-------------------- **Solution** ----------------------------

The specific heat depends on the temperature as in the figure. At high temperature the value of C_v is close to 3R



In classical theory the average energy is

$$\bar{\varepsilon} = KT \qquad (1)$$

And the energy per mole is

$$U = 3N_A KT = 3RT \qquad (2)$$

So the specific heat at constant volume is

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 3R \qquad (3)$$

This is in agreement with experiment at high temperature, but it fails completely at low temperatures.

Q2. (a) Differentiate between bosons and fermions.

--------------------- Solution ----------------------------

fermions	Boson
Anti-Symmetric wave function	Symmetric wave function
Odd atoms - Protons - electrons	Even atoms - photons
Fermi Dirac statistics	Bose-Einstein statistics

Q2 (b) Discus in details the black body radiation phenomena

--------------------- Solution ----------------------------

The Black body radiations can be considered as the photon gas. Photons are taken as bosons and they are obey BE statistics

$$n_i = \frac{g_i}{e^{\varepsilon_i lKT} - 1} \qquad (1)$$

We can write

$$dn = \frac{g(\nu)d\nu}{e^{h\nu lKT} - 1} \qquad (2)$$

The translational kinetic energy for a particle in a cubical box is

$$\varepsilon = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2) \tag{3}$$

In gama space

$$r^2 = n_x^2 + n_y^2 + n_z^2 \tag{4}$$

$$r^2 = \frac{8mL^2}{h^2}\varepsilon \tag{5}$$

So

$$gd\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon \tag{6}$$

Since the photon has no rest mass, so we can write

$$gd\nu = \frac{4\pi V}{h^3}\frac{h^2\nu^2}{c^2}\frac{h}{c}d\nu \tag{7}$$

The energy per unit volume, energy density, is

$$\rho d\nu = \frac{dn}{V}h\nu \tag{8}$$

So

$$\rho d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/KT} - 1}d\nu \tag{9}$$

Which represents Planck's radiation law

Q3. Find the thermodynamic function (C_V only) for the electron gas.

-------------------- Solution ----------------------------

For the electron gas

$$U = \frac{3}{5}N\varepsilon_F\left[1 + \frac{5\pi^2}{12}\left(\frac{KT}{\varepsilon_F}\right)^2 - \dots\right] \tag{1}$$

The heat capacity at constant volume C_V is

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{\pi^2}{2}\frac{KT}{\varepsilon_F} NK \qquad (2)$$