Faculty of Science,
Phys. Department.
Phys. 414 Electric Circuit

Time: 2 hr.
Final Exam

Answer

1- The circuit has four branches and three nodes.

2-

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \qquad (2.8.1)$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \qquad (2.8.2)$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \qquad (2.8.3)$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \quad \Longrightarrow \quad v_3 = v_2 \qquad (2.8.4)$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \quad \Longrightarrow \quad i_3 = \frac{i_2}{2} \qquad (2.8.5)$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \quad i_3 = 1 \text{ A}, \quad v_1 = 24 \text{ V}, \quad v_2 = 6 \text{ V}, \quad v_3 = 6 \text{ V}$$

3-

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

and

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

and

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

4-

At node 1,

$$3 = i_1 + i_x \quad \Longrightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \qquad (3.2.1)$$

At node 2,

$$i_x = i_2 + i_3 \qquad \Longrightarrow \qquad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \qquad (3.2.2)$$

At node 3,

$$i_1 + i_2 = 2i_x \qquad \Longrightarrow \qquad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1 - 3v_2 + v_3 = 0 \qquad (3.2.3)$$

By Cramer' method

Thus, we find

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ V}, \qquad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ V}$$

5-

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad (2.6.1)$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad (2.6.2)$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \qquad \Longrightarrow \qquad i = -8 \text{ A}$$

and $v_o = 48$ V.

6-

Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \qquad \Longrightarrow \qquad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \qquad \Longrightarrow \qquad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

$$i_o = i_2 = \frac{12}{76} \text{ A}$$

7-

The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

$$i_o = i'_o + i''_o \qquad (4.4.1)$$

where i'_o and i''_o are due to the 4-A current source and 20-V voltage source respectively. To obtain i'_o, we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i'_o. For loop 1,

$$i_1 = 4 \text{ A} \qquad (4.4.2)$$

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \tag{4.4.3}$$

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \tag{4.4.4}$$

But at node 0,

$$i_3 = i_1 - i'_o = 4 - i'_o \tag{4.4.5}$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i'_o = 8 \tag{4.4.6}$$

$$i_2 + 5i'_o = 20 \tag{4.4.7}$$

which can be solved to get

$$i'_o = \frac{52}{17}\text{ A} \tag{4.4.8}$$

To obtain i''_o, we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i''_o = 0 \tag{4.4.9}$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_o = 0 \tag{4.4.10}$$

But $i_5 = -i''_o$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i''_o = 0 \tag{4.4.11}$$

$$i_4 + 5i''_o = -20 \tag{4.4.12}$$

which we solve to get

$$i''_o = -\frac{60}{17}\text{ A}$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706\text{ A}$$

8-

Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \quad \Longrightarrow \quad i_o = 6\text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24\text{ V}$$