جامعة بنها
كلية العلوم – قسم الفيزياء
دور يناير ٢٠١٤

الفرقة الأولي
مادة: فيزياء عامة (١)
الزمن ٣ ساعات

(تخلفات)

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تاريخ الامتحان: ٢٠١٤/١٢/٢١

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٤. (أ) استنتج معادلة جاوس في العدسات

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Consider a lens having an index of refraction n and two spherical surfaces of radii of curvature R_1 and R_2, as in Fig. (1). An object is placed at point O at a distance p_1 in front of surface 1. For this example, p_1 has been chosen so as to produce a virtual image I_1 to the left lens. This image is then used as the object for surface 2, which results in a real image I_2.

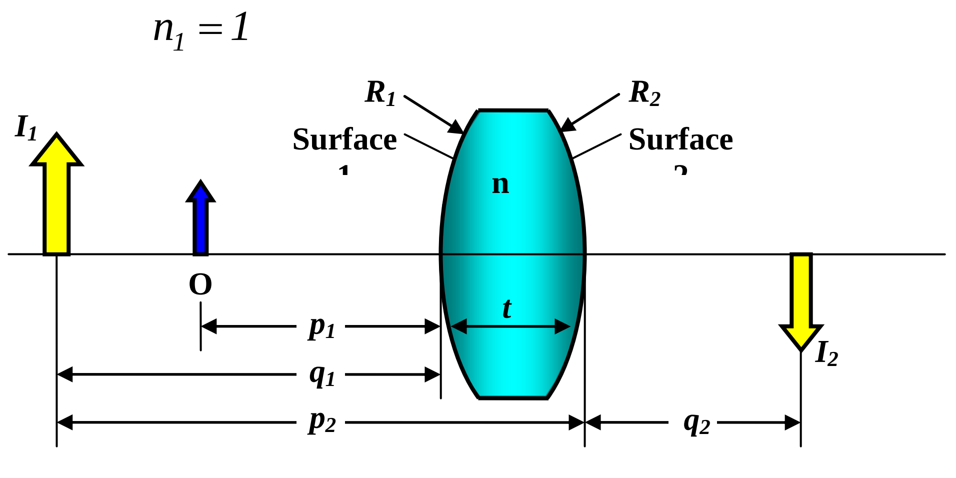

Fig. (1): To locate the image of a lens, the image at I_1 formed by the first surface is used as the object for the second surface. The final image is at I_2

Using Eq. (??) and assuming $n_1 = 1$ because the lens is surrounded by air, we find that the image formed by surface 1 satisfies the equation

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}. \tag{1}$$

Now we apply Eq. (??) to surface 2, taking $n_1 = n$ and $n_2 = 1$. That is, light approaches surface 2 as if it had come from I_1. Taking p_2 as the object distance and q_2 as the image distance for surface 2 gives

$$\frac{1}{p_2} + \frac{n}{q_2} = \frac{1-n}{R_2}. \tag{2}$$

But $p = -q_1 + t$, where t is the thickness of the lens. (Remember q_1 is a negative number and p_2 must be positive by our sign convention.) For a thin lens, we can neglect *t*. In this approximation and from Fig. (1), we see that $p_2 = -q_1$. Hence, Eq. (2) becomes

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}. \tag{3}$$

Adding Eqs. (1) and (3), we find that

$$\frac{1}{p_1}+\frac{1}{q_2}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right). \qquad (4)$$

For the thin lens, we can omit the subscripts on p_1 and q_2 in Eq. (4) and call the object distance p and the image distance q, as in Fig. (2). Hence, we can write Eq. (4) in the form

$$\frac{1}{p}+\frac{1}{q}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right). \qquad (5)$$

This equation relates the image distance q of the image formed by a thin lens to the object distance p and to the thin lens properties (index of refraction and radii of curvature). It is valid only for paraxial rays and only when the lens thickness is small relative to R_1 and R_2.

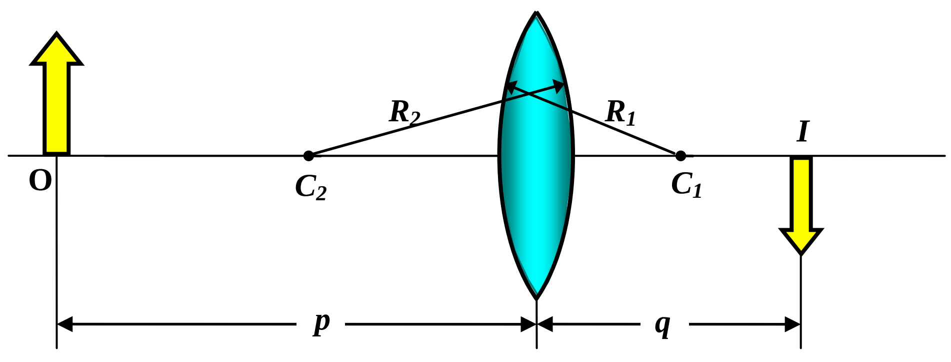

Fig. (2): The biconvex lens.

We now define the focal length f of a thin lens as the image distance that corresponds to an infinite object distance, as we did with mirrors. According to this definition and from Eq. (5), we see that as $p \to \infty,\ q \to f$; therefore, the inverse of the focal length for a thin lens is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{6}$$

This equation is called the ***lens makers' equation*** because it enables f to be calculated from the known properties of the lens. It can be used to determine the values of R_1 and R_2 needed for a given index of refraction and desired focal length.

Using Eq. (6), we can write Eq. (5) in an alternate form identical to Eq. (??) for mirrors:

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.} \tag{7}$$

A thin lens has two focal points, corresponding to incident parallel light rays traveling from the left or right. This is illustrated in Fig. (3) for biconvex lens (converging, positive f)

and a biconcave lens (diverging, negative f). Table (1) gives the complete sign conventions for lenses.

Table (1): Sign convention for lenses

p is + if the object is in front of the lens.
p is - if the object is in back of the lens.
q is + if the image is in back of the lens.
q is - if the image is in front of the lens.
R_1 and R_2 are + if the center of curvature is in back of the lens.
R_1 and R_2 are - if the center of curvature is in front of the lens.

Note that the sign conventions for thin lenses are the same for refracted surfaces. Applying these rules to a converging lens, we see that when $p > f$, the quantities *p*, *q*, and R_1 are positive and R_2 is negative. Therefore, when a converging lens forms a real image from a real object, *p*, *q*, and *f* are all positive. For a diverging lens, *p* and R_2 are positive, *q* and R_1 are negative, and so *f* is negative for a divergence lens.

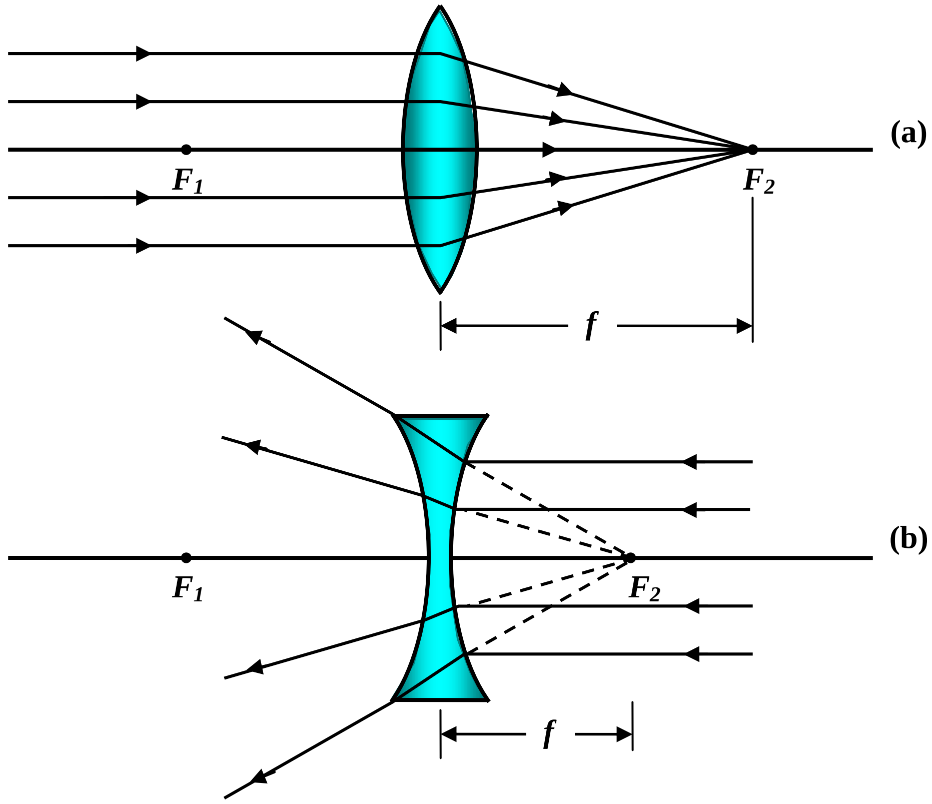

Fig. (3): The object and image focal point of (a) the biconvex lens and (b) the biconcave lens.

٥. (أ) ارسم مسارات الأشعة فى الميكروسكوب المركب واستنتج معامل التكبير الكلى.

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Greater magnification can be achieved by combining two lenses in a device called a compound microscope, a schematic diagram of which is shown in Fig. (7). It consists of an objective lens that has a very short focal length $f_o < 1cm$, and an eyepiece lens having a focal length, f_e, of a few centimeters. The two lenses are separated by a distance L, where L is much greater than either f_o or f_e. The object, which is placed just to the left of the focal point of the objective, forms a real, inverted image at I_1, which is at or close to the focal point of the eyepiece. The eyepiece, which serves as a simple magnifier, produces at I_2 an image of the image at I_1, and this image at I_2 is virtual and inverted. The lateral magnification, M_1, of the first image is $-q_1 / p_1$. Note from Fig. (7) that q_1 is approximately equal to L, and recall that the object is very close to the focal point of the objective; thus, $p_1 \approx f_o$. This gives a magnification for the objective of

$$M_1 \approx -\frac{L}{f_o}$$

The angular magnification of the eyepiece for an object (corresponding to the image at I_1) placed at the focal point of the eyepiece is found from Eq. (5) to be

$$m_e = \frac{25}{f_e}$$

The overall magnification of the component microscope is defined as the product of the lateral and angular magnifications:

$$M = M_1 m_e = -\frac{L}{f_o}\left(\frac{25}{f_e}\right). \tag{6}$$

The negative sign indicates that the image is inverted.

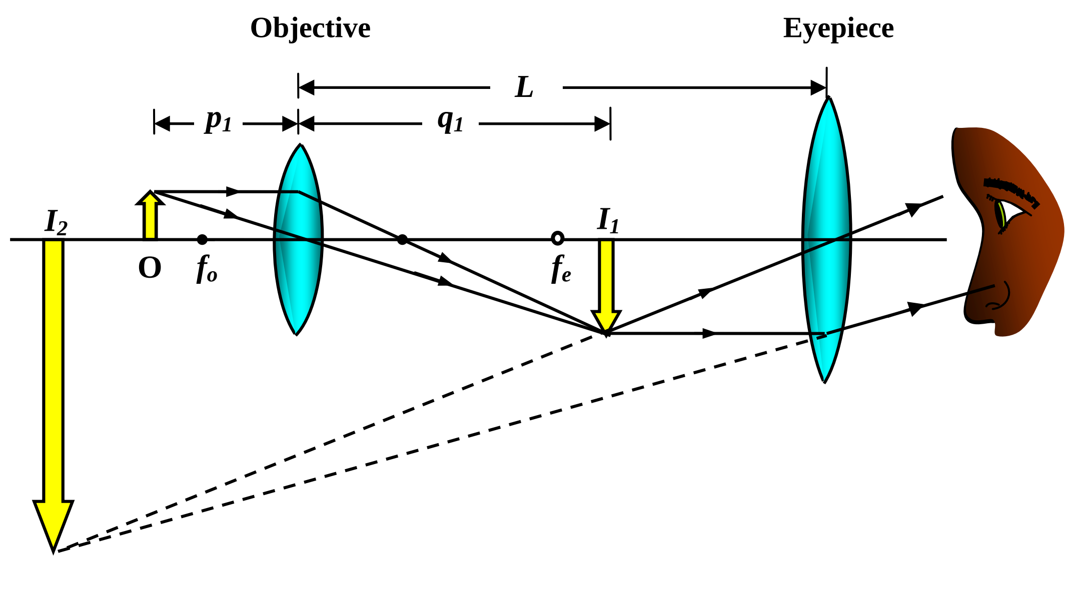

Fig. (7): Diagram of the compound microscope

٦. (أ) استنتج معادلة جاوس فى المرايا.

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We can us the geometry shown in Fig. (5) to calculate the image distance q from a knowledge of the object distance p and the mirror radius of curvature, R. By convention, these distances are measured from point V. We assume that the object at point O. Therefore, any ray leaving O is reflected at the spherical surface and focus at a point I, the image point. Let us proceed by considering the geometric construction in Fig. (5), which shows a single ray leaving point O and focusing at point I.

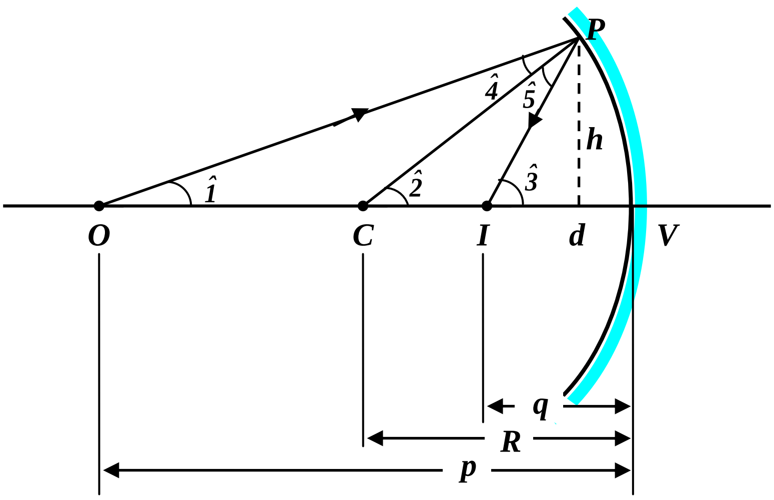

Fig. (5): The image formed by a spherical concave mirror where the object *O* lies outside the center of curvature, *C*.

We assume the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this to the triangles OPC and OPI gives:

$$\hat{2} = \hat{1} + \hat{4}, \tag{1}$$

$$\hat{3} = \hat{2} + \hat{5}. \tag{2}$$

From Eqs. (1) and (2)

$$\hat{4} = \hat{2} - \hat{1} \qquad \text{and} \qquad \hat{5} = \hat{3} - \hat{2}$$

From the law of reflection $\hat{4} = \hat{5}$. So

$$\hat{2} - \hat{1} = \hat{3} - \hat{2}$$

or

$$\hat{3} + \hat{1} = 2(\hat{2}). \tag{3}$$

From the triangles in Fig. (5), we note that:

$$\tan\hat{1} = \frac{h}{p-d}, \quad \tan\hat{3} = \frac{h}{q-d}, \quad \text{and} \quad \tan\hat{2} = \frac{h}{R-d}$$

The substitution in Eq. (3) gives

$$\frac{h}{p-d} + \frac{h}{q-d} = 2\left(\frac{h}{R-d}\right)$$

or

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{2}{R}} \tag{4}$$

This equation is called the ***mirror equation***. If the object is very far from the mirror, p can be said to approach infinity, then

$\frac{1}{p} \approx 0$, and we see from Eq. (6) that $q \approx \frac{R}{2}$. That is, when the object is very far from the mirror, the image point is halfway between the center of the curvature and the center of the mirror, as in Fig. (6). The rays are essentially parallel in this figure and we call the image point in this special case the ***focal point***, *F*, and the image distance the ***focal length***, *f*, where

$$f = \frac{R}{2}. \tag{5}$$

The mirror equation can be expressed in terms of the focal length:

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.} \tag{6}$$

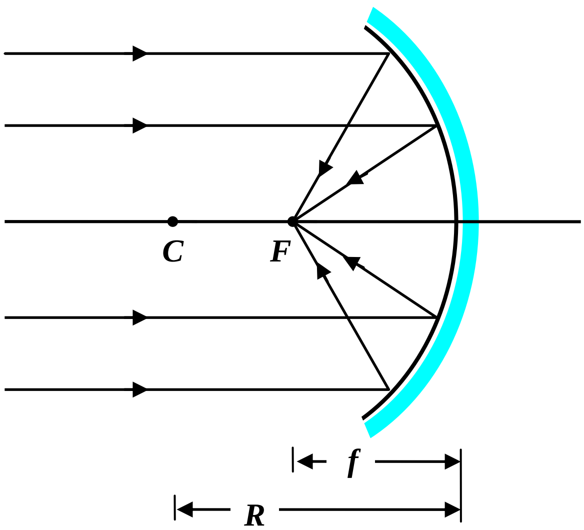

Fig. (6): Light rays from infinity reflect from a concave mirror through the focal point *F*.

٦. (ب) كيف يمكنك تصميم مجموعة من عدستين منفصلتين خالية تماما من الزيغ اللوني

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It is possible to make an achromatic combination of two lenses of the same material and separated by a finite distance. Suppose two lenses of focal lenses f_1, f_2 and mean refractive index n are situated a distance x apart. The equivalent focal length of the combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}, \qquad (1)$$

or

$$P = P_1 + P_2 - xP_1P_2 , \tag{2}$$

where

$$P_1 = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \tag{3}$$

$$P_2 = (n-1)\left(\frac{1}{R_3} - \frac{1}{R_4}\right), \tag{4}$$

By partial differentiation of Eqs. (2) and (4), we get

$$dP_1 + dP_2 - xP_1\, dP_2 - xP_2\, dP_1 = 0 , \tag{5}$$

and

$$dP_1 = \frac{P_1}{n-1} dn \quad \text{and} \quad dP_2 = \frac{P_2}{n-1} dn , \tag{6}$$

By substituting from Eq. (6) in Eq. (5), we get

$$\frac{P_1}{(n-1)} dn + \frac{P_2}{(n-1)} dn - \frac{xP_2P_1}{(n-1)} dn - \frac{xP_2P_1}{(n-1)} dn = 0$$

or

$$P_1 + P_2 - 2xP_1P_2 = 0 , \tag{7}$$

From the last equation we get

$$x = \frac{1}{2}\left[\frac{1}{P_1} + \frac{1}{P_2}\right] = \frac{f_1 + f_2}{2} . \tag{8}$$

Thus, we get an achromatic combination if the two lenses are separated by a distance equal to half the sum of their focal lengths.