Benha University
Faculty of Science
Math. Dept.

Model Answer for

Algebra

Date of exam: 31 / 12 / 2018

Dr. Salah Gomaa Elgendi

نموذج اجابه لمادة الجبر

ورقة كاملة

تاريخ الامتحان: 31 - 12 -2018

دكتور/ صلاح جمعة الجندي

Total: 80 points

Q1) **(15 points)**

Step 1. For a positive integer $n = 1$,

$L.H.S. = \frac{1}{2} = R.H.S.$,

$\therefore$ *the relation is true for* $n = 1$.

Step 2. Suppose that the relation is true for $n = k$.

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^k} = 1 - \frac{1}{2^k} \qquad (3)$$

Step 3. We want to prove that the relation is true for $n = k + 1$

$L.H.S. = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$

by substituting from Eq. (3)

$L.H.S. = 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$= 1 - \frac{1}{2^k}(1 - \frac{1}{2})$

$= \frac{1}{2^{k+1}}$

$= R.H.S.$ at $n = k + 1$.

Then, the relation is true for $n = k + 1$.

Then, it is true for all positive integers $n \geq 1$.

Then, the relation is true for $n = k + 1$. Hence, it is true for all positive integers $n \geq 1$.

Q2)a) **(10 points)**

This polynomial function is in standard form, however it is missing two terms. We can rewrite the function as $f(x) = x^5 + 0x^4 - 2x^3 + 7x^2 + 0x - 11$ to fill in the missing terms.

-3	1	0	-2	7	0	-11
		-3	9	-21	42	-126
	1	-3	7	-14	42	$\boxed{-137}$

With the remainder theorem, we can find this value much quicker.

$$r = f(-3) = (-3)^5 - 2(-3)^3 + 7(-3)^2 - 11 = \boxed{-137}$$

b) **(10 points)**

Any square matrix A can be written as the sum of the symmetric matrix R and the skew symmetric matrix S where

$$R = \frac{1}{2}(A + A^t) \text{ and } S = \frac{1}{2}(A - A^t).$$

$$R = \frac{1}{2}\begin{pmatrix} 2 & 6 & 3 \\ 6 & 12 & 3 \\ 3 & 3 & 4 \end{pmatrix} \ \& \ S = \frac{1}{2}\begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & -3 \\ -3 & 3 & 0 \end{pmatrix}.$$

Q3) a) **(15 points)**

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 0 \\ 0 & 3 & 2 \end{bmatrix}$ and $det(A) = 22 \neq 0$, then the matrix A is non singular.

To find A^{-1} let's calculate the following:

$$C(A) = \begin{pmatrix} \mathbf{2} & \mathbf{-8} & \mathbf{12} \\ \mathbf{5} & \mathbf{2} & \mathbf{-3} \\ \mathbf{-3} & \mathbf{3} & \mathbf{-7} \end{pmatrix} \quad and \quad adj(A) = \begin{pmatrix} \mathbf{2} & \mathbf{5} & \mathbf{-3} \\ \mathbf{-8} & \mathbf{2} & \mathbf{3} \\ \mathbf{12} & \mathbf{-3} & \mathbf{-7} \end{pmatrix}$$

Then,
$$A^{-1} = \frac{1}{22}\begin{pmatrix} \mathbf{2} & \mathbf{5} & \mathbf{-3} \\ \mathbf{-8} & \mathbf{2} & \mathbf{3} \\ \mathbf{12} & \mathbf{-3} & \mathbf{-7} \end{pmatrix}$$

b) **(10 points)**

$$\frac{1}{(2+x)^3} = (2+x)^{-3} = 2^{-3}\left(1+\frac{x}{2}\right)^{-3} = 2^{-3}\left[1-3\left(\frac{x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{x}{2}\right)^3+\dots\right]$$

$$= \frac{1}{2^3}\left[1-\frac{3}{2}x+\frac{3}{2}x^2-\frac{5}{4}x^3+\dots\right]$$

$$= \frac{1}{8}\left[1-\frac{3}{2}x+\frac{3}{2}x^2-\frac{5}{4}x^3+\dots\right]$$

The series is true provided $\left|\frac{x}{2}\right| < 1$ i.e. $|x| < 2$

Q4)a) **(10 points)**

Since $2+3i$ is an root then $2-3i$ is an root and hence by Vieta equations

$$(2+3i)+(2-3i)+a+b=-(-3)=3$$

$\therefore a+b=-1$

$$(2+3i)\cdot(2-3i)\cdot a\cdot b=-26$$

$13ab=-26 \Rightarrow \therefore ab=-2$

By substituting

$$a(-1-a)=-2 \Rightarrow (a+2)a-1)=0$$

$$a=1 \quad \text{or} \quad \therefore a=-2$$

$$b=1 \quad \text{or} \quad \therefore b=-2$$

Then the roots are

$$(2+3i),(2-3i),-2,1$$

b) **(10 points)**

$$\frac{8x-42}{x^2+3x-18}=\frac{8x-42}{(x+6)(x-3)}=\frac{A}{(x+6)}+\frac{B}{(x-3)}$$

$$\therefore 8x-42=A(x-3)+B(x+6)$$

By using substituting method,

$x=3 \Rightarrow -18=9B \Rightarrow B=-2,$

$x=-6 \Rightarrow -90=-9A \Rightarrow A=10,$

$$\therefore \frac{8x-42}{x^2+3x-18}=\frac{10}{(x+6)}-\frac{2}{(x-3)}$$

With best wishes

Dr. Salah Gomaa Elgendi