جامعة بنها – كلية العلوم – قسم الرياضيات

# لطلاب المستوى الثانى

ترم تخرج

يوم الامتحان: الاحد    19 / 5 / 2019 م

# المادة : رياضيات متقطعة   (225 ر)

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مدرس بقسم الرياضيات بكلية العلوم

اسئله + نموذج إجابه

ورقة كاملة

## رياضيات متقطعة (225 ر) لطلاب المستوى الثانى

**Answer the following questions: (80 marks)** أجب على الاسئله التاليه (الدرجة الكلية 80 درجة)

## Question 1. السؤال الأول (30 درجة) :-

1. **Define** a Boolean algebra $(B, \oplus, *, ^{-}, 0, 1)$ and for all $b_1, b_2 \in B$, **prove that**: There is only one element $\overline{b_1} \in B$ such that $b_1 \oplus \overline{b_1} = 1$ and $b_1 * \overline{b_1} = 0$ .
2. For any propositions $p, q, r$, **Prove that:**
   - I. $\overline{(p \veebar q)} \equiv (p \veebar \overline{q})$,
   - II. $[(p \to q) \wedge (p \vee r)] \vdash (q \vee r)$ .
3. A relation $\equiv_5$ on the set $Z$ is defined by $a \equiv_5 b$ if and only if $a - b = 5k$ for some $k \in Z$ , **show that** $\equiv_5$ is an equivalent relation and **describe** the equivalence classes [3], [-1].

## Question 2. السؤال الثانى (30 درجة)

**1-** Let $A$ , $B$ , $C$, $D$ be sets, **prove that:**

- I. $P(A) \cap P(B) = P(A \cap B)$
- II. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- III. $A \cup (B - C) = (A \cup B) - (\overline{A} \cap C)$

2- **draw** diagram to represent the graph whose adjacency matrix is given below. **Write down** the degree of each vertex, and **state** the graph is (a) *simple* ; (b) *regular* ; (c) *Eulerian*.?

$$A = \begin{pmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 1 & 3 & 0 & 1 \\ 0 & 3 & 1 & 2 & 1 \\ 1 & 0 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{pmatrix}$$

3- **Design** a logic network for the following so that the output is described by the following Boolean expression: $x_1 x_3 \oplus \overline{x_1} \oplus x_2 \overline{x_3}$.

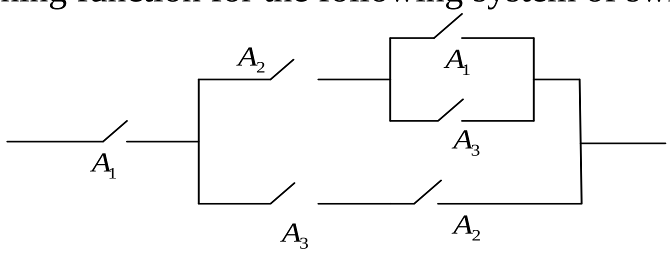
## Question 3. السؤال الثالث (20 درجة):

1. **Define** *the complete graph* $K_n$ , *the complete bipartite graph* $K_{r,s}$ and *Eulerian path*, and for **which values** of $n, r, s$, the graphs $K_n$ , $K_{r,s}$ are **Eulerian**?

2. **Show** the following function is a bijection and **find** its inverse:

$$f: R \to R, f(x) = (5x - 3)^3 \forall x \in R.$$

3. **Define** a switching function for the following system of switches:




انتهت أسئلة

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مع أطيب تمنياتى بالتوفيق والنجاح
د. محمد السيد عبدالعال

**Good Luck !**

## نموذج اجابه لأمتحان رياضيات متقطعة (225 ر) لطلاب المستوى الثانى

## (الدرجة الكلية 80 درجة )

### اجابة السؤال الأول (30 درجة) :-

1. **Define** a Boolean algebra $(B, \oplus, *, ^{-}, 0, 1)$ and for all $b_1, b_2 \in B$, **prove that:** There is only one element $\overline{b_1} \in B$ such that $b_1 \oplus \overline{b_1} = 1$ and $b_1 * \overline{b_1} = 0$ .

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**Boolean algebra** consists of a set *B* together with three operations defined on that set. These are:
(a) a binary operation denoted by ⊕ referred to as the **sum** ;
(b) a binary operation denoted by ∗ referred to as the **product** ;
(c) an operation which acts on a single element of *B*, denoted by -, where, for any element $b \in B$, the element $b^- \in B$ is called the **complement** of $b^-$ (An operation which acts on a single member of a set *S* and which results in a member of *S* is called a **unary operation**.)
The following axioms apply to the set *B* together with the operations ⊕, ∗ and - .
B1. Distinct identity elements belonging to *B* exist for each of the binary operations ⊕ and ∗ and we denote these by **0** and **1** respectively. Thus we have

$$b \oplus \mathbf{0} = \mathbf{0} \oplus b = b$$
$$b * \mathbf{1} = \mathbf{1} * b = b \quad \text{for all } b \in B.$$

for all $a, b, c \in B$.

$$(a * b) * c = a * (b * c)$$
$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

B2. The operations ⊕ and ∗ are associative, that is
B3. The operations ⊕ and ∗ are commutative, that is

$$a \oplus b = b \oplus a$$

$$a * b = b * a \quad \text{for all } a, b \in B.$$

B4. The operation ⊕ is distributive over ∗ and the operation ∗ is distributive over ⊕, that is

$$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$$

$$a * (b \oplus c) = (a * b) \oplus (a * c) \text{ for all } a, b, c \in B.$$

B5. For all b ∈ B, b ⊕ .b = 1 and b ∗ .b = 0.

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$(b_1 \oplus b_2) \oplus (\overline{b_1} * \overline{b_2}) = [(b_1 \oplus b_2) \oplus \overline{b_1}] * [(b_1 \oplus b_2) \oplus \overline{b_2}]$ (axiom B4)

$= [\overline{b_1} \oplus (b_1 \oplus b_2)] * [(b_1 \oplus b_2) \oplus \overline{b_2}]$ (axiom B3)

$= [(\overline{b_1} \oplus b_1) \oplus b_2] * [b_1 \oplus (b_2 \oplus \overline{b_2})]$ (axiom B2)

$= (1 \oplus b_2) * (b_1 \oplus 1)$ (axiom B5)

$= 1 * 1$ (theorem 9.4)

$= 1$ (axiom B1).

We have proved that $(b_1 \oplus b_2) \oplus \overline{b_1} * \overline{b_2}) = 1$ so that $\overline{b_1} * \overline{b_2}$ is the complement of $b_1 \oplus b_2$, i.e. $\overline{(b_1 \oplus b_2)} = \overline{b_1} * \overline{b_2}$.

That $\overline{(b_1 * b_2)} = \overline{b_1} \oplus \overline{b_2}$ follows from the duality principle.

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2. For any propositions $p, q, r$, **Prove that:**
   I. $(\overline{p \veebar q}) \equiv (p \veebar \overline{q})$,
   II. $[(p \to q) \wedge (p \vee r)] \vdash (q \vee r)$ .

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I. $(\overline{p \veebar q}) \equiv (p \veebar \overline{q})$

| p | q | $\overline{q}$ | $p \veebar q$ | $\overline{p \veebar q}$ | $p \veebar \overline{q}$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 |

I. $[(p \to q) \wedge (p \vee r)] \vdash (q \vee r)$ .

| p | q | r | $p \to q$ | $p \vee r$ | $(p \to q) \wedge (p \vee r)$ | $q \vee r$ | $[(p \to q) \wedge (p \vee r)] \to (q \vee r)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |

3. A relation $\equiv_5$ on the set $Z$ is defined by $a \equiv_5 b$ if and only if $a-b=5k$ for some $k \in Z$, **show that** $\equiv_5$ is an equivalent relation and **describe** the equivalence classes [3], [-1].

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In this case $a \equiv_5 b$ if and only if a − b = 5k for some integer k; that is, if and only if there exists an integer k such that a = 5k + b.
Firstly, R is reflexive since a-a = 5 0,
Secondly, if $a \equiv_5 b$ i.e. a-b =5k then b-a = −5k so implies $b \equiv_5 a$ therefore $\equiv_5$ is symmetric.
Thirdly, suppose $a \equiv_5 b$ and $b \equiv_5 c$; then there exist integers k such that a-b =5k and b-c =5$k_1$.
Combining these two equations gives a-c =5(k-$k_1$) therefore $a \equiv_5 c$
where (k-k1)is an integer. Thus $a \equiv_5 b$ and $b \equiv_5 c$ implies $a \equiv_5 c$ so $\equiv_5$ is transitive.

Therefore [p] = {q ∈ z : q = 5k + p, for some k ∈ z}.

[3] = {q ∈ z : q = 5k + 3, for some k ∈ z}.

[-1] = {q ∈ z : q = 5k + -1, for some k ∈ z}.

**Question 2.**

I. $P(A) \cap P(B) = P(A \cap B)$

Let $X \in P(A) \cap P(B)$. Then $X \in P(A)$ and $X \in P(B)$. Hence $X \subseteq A$ and either $X \subseteq B$ Therefore $X \subseteq A \cap B$. It follows that $X \in P(A \cap B)$
We have shown that if $X \in P(A) \cap P(B)$ then $X \subseteq P(A \cap B)$ Therefore $P(A) \cap P(B) \subseteq P(A \cap B)$
Secondly we must show that $P(A \cap B) \subseteq P(A) \cap P(B)$
Let $X \in P(A \cap B)$ Then $X \subseteq A \cap B$ so $X \subseteq A$ and $X \subseteq B$ Hence $X \in P(A)$ and either $X \in P(B)$. Therefore $X \in P(A) \cap P(B)$.
Therefore $(P(A \cap B) \subseteq P(A) \cap P(B)$

Finally, since we have shown that each set is a subset of the other, we may conclude
$P(A)\cap P(B)=P(A\cap B)$

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II. $A\times(B\cap C)=(A\times B)\cap(A\times C)$

Let (a, x) ∈ A × (B ∩ C ). By the definition of the Cartesian product, this means that a ∈ A and x ∈(B ∩ C ). Thus x ∈ X, so (a, x) belongs to A× B; and x ∈ C , so (a, x) belongs to A×C as well. Therefore (a, x) ∈ (A× B)∩(A×C ), which proves that A × (B ∩ C ) ⊆ (A × B) ∩ (A × C ).

To prove the subset relation the other way round as well, let
(a, x) ∈ (A × B) ∩ (A × C ).

Then (a, x) ∈ (A × B), so a ∈ A and x ∈ B; and (a, x) ∈ (A × C ), so a ∈ A and x ∈ C . Therefore a ∈ A and x ∈ (B ∩ C ) which means that the ordered pair (a, x) belongs to the Cartesian product A × (B ∩ C ). Hence (A × B) ∩ (A × C ) ⊆ A × (B ∩ C ).

The conclusion that the sets A × (B ∩ C ) and (A × B) ∩ (A × C ) are equal now
Then $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

III. $A\cup(B-C)=(A\cup B)-(\bar{A}\cap C)$

$$A\cup(B-C)=A\cup(B\cap\bar{C})=(A\cup B)\cap(A\cup\bar{C})=(A\cup B)-\overline{(A\cup\bar{C})}$$

$$=(A\cup B)-\overline{(A\cup\bar{C})}=(A\cup B)-(\bar{A}\cap C)$$

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1. **draw** diagram to represent the graph whose adjacency matrix is given below. **Write down** the degree of each vertex, and **state** the graph is (a) *simple* ; (b) *regular* ; (c) *Eulerian*.?

$$A=\begin{pmatrix}1&2&0&1&1\\2&1&3&0&1\\0&3&1&2&1\\1&0&2&1&2\\1&1&1&2&1\end{pmatrix}$$

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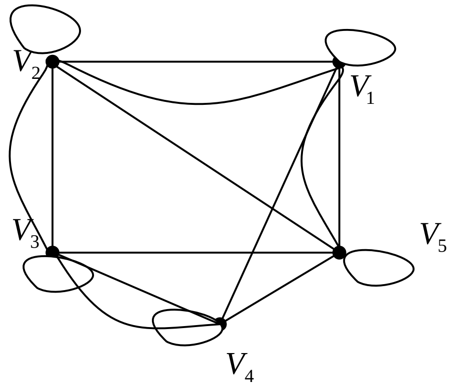


the graph is not *simple* ; not *regular* ; not *Eulerian*

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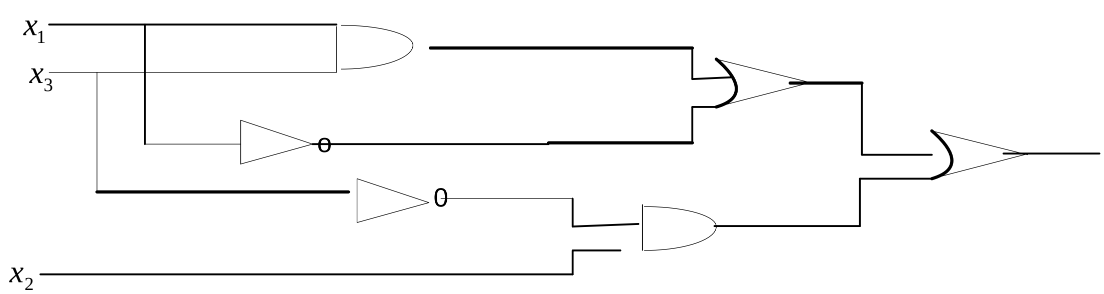
2. **Design** a logic network for the following so that the output is described by the following Boolean expression: $x_1 x_3 \oplus \overline{x_1} \oplus x_2 \overline{x_3}$.




## أجابة السؤال الثالث (20 درجة) :-

1. **Define** *the complete graph* $K_n$ *, the complete bipartite graph* $K_{r,s}$ and *Eulerian path*, and for **which values** of *n, r, s,* the graphs $K_n$ , $K_{r,s}$ are **Eulerian**?

## الحـــــــــــــــل

A graph is connected if, given any pair of distinct vertices, there exists a path connecting them.
A graph in which every vertex has the same degree r is called regular (with degreer) or simply r-regular.
A graph is Eulerian if the sum of all entry in any row or in any column of its adjacency matrix is even.
A complete graph is a simple graph in which every pair of distinct vertices is joined by an edge.
A complete bipartite graph is a bipartite graph such that every vertex of $V_1$ is joined to every vertex of $V_2$ by a unique edge.

An Eulerian path in a graph G is a closed path which includes every edge of G . A graph is said to be Eulerian if it has at least one Eulerian path.

The complete graph $K_n$ is (n−1)-regular−every vertex has degree n−1. Since it is connected, $K_n$ is Eulerian if and only if n is odd (so that n − 1is even).

A complete bipartite graph $K_{r,s}$ is Eulerian if and only if $r,s$ is even.

2. **Show** the following function is a **bijection** and find its **inverse**:

$$f : R \to R,\ f(x) = (5x-3)^3 \ \forall x \in R.$$

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To show that *f* is an injection we prove that, for all real numbers *x* and *y*, *f (x) = f (y)* implies *x = y*. Now *f (x) = f (y)*

$(5x-3)^3 = .(5y-3)^3 \Rightarrow x = y$ so *f* is injective.

To show that *f* is a surjection, let y be any element of the codomain f . We need to find x ∈ R such that f (x) = y. Let $x = \frac{\sqrt[3]{y}+3}{5}$. Then x ∈ R and

$f(x) = [5\frac{\sqrt[3]{y}+3}{5} - 3]^3 = y$ so f is surjective.

To find $f^{-1}$ we simply use its definition: if y = f (x) then x = $f^{-1}$(y).

Now y = f (x)

$\Rightarrow y = (5x-3)^3 \Rightarrow x = \frac{\sqrt[3]{y}+3}{5}$

$x = f^{-1}(y) = \frac{\sqrt[3]{y}+3}{5}$. Therefore the inverse function is $f^{-1} : R \to R,\ f^{-1}(y) = \frac{\sqrt[3]{y}+3}{5}$.

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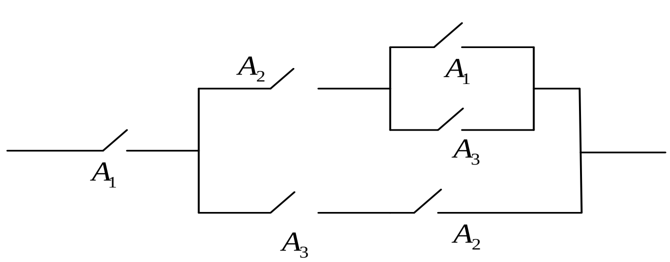
3. **Define** a switching function for the following system of switches:




$f(x_1, x_2, x_3) = x_1[x_2(x_1 \oplus x_3) \oplus x_3x_2]$