جامعة بنـها – كلية العلوم – قسم الرياضيات

لطلاب المستوى الثانى

يوم الامتحان: الاحد 13 / 1 / 2019 م

المادة : رياضيات متقطعة (225 ر)

الممتحن: د . / محمد السيد عبدالعال عبدالغنى

مدرس بقسم الرياضيات بكلية العلوم

اسئله + نموذج إجابه

ورقة كاملة

رياضيات متقطعة (225 ر) لطلاب المستوى الثانى

أجب على الاسئله التاليه (الدرجة الكلية 80 درجة)

السؤال الأول (25 درجة) :-

1- **Show** the following function is a **bijection** and find its **inverse**:

$$f: R-\{5\} \to R-\{2\},\ f(x)=\frac{2x+1}{x-5}\ \forall x \in R-\{5\}.$$

2- Let $A=\{1,2,3,4,5\}$ and $R=\{(a,b): b \le a^2\}$ be a relation on *A*. **List** the elements of *R*, and **write** down its binary matrix. **Determine** which of the properties, reflexive, symmetric, transitive the relation *R* is satisfied.

3- For all propositions *p*, *q*, *r*, **Prove that:**

I. $(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv \overline{p}$ ***by laws***

II. $[(p \vee q) \to \overline{r}]\ \underline{\vee}\ (\overline{p} \vee \overline{q})$.

4- draw the diagram of a system of switches for which it is the switching function:

$f(x_1, x_2, x_3) = x_1[x_2(x_1 \oplus x_3) \oplus x_3x_2]$

السؤال الثانى (30 درجة) :-

1- Let $A=\{1,2,3\}$, $B=\{1,2\}$ and $P(A)$ is the power set of A. **Determine** whether each of the following is true or false and give a brief justification.

$(i)\ B \in P(A)$, $(ii)\ B \in A$, $(iii)\ B \subseteq P(A)$, , $(iv)\ A \subseteq P(A)$.

2- **Let** $(B, \oplus, *, \bar{\ }, 0, 1)$ be a Boolean algebra and for all $b_1, b_2 \in B$, **prove that**:

There is only one element $\overline{b_1} \in B$ such that $b_1 \oplus \overline{b_1} = 1$ and $b_1 * \overline{b_1} = 0$.

3- **Let** *A* , *B* , *C* are sets, **prove that:**

I. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

II. $A-(B \cap C)=(A-B) \cup (A-C)$ ***by definitions***

4- **Defined** *the complete graph* K_n , *the complete bipartite graph* $K_{r,s}$ and *Eulerian path*, and for **which values** of n, r, s , the graphs K_n , $K_{r,s}$ are **Eulerian**?

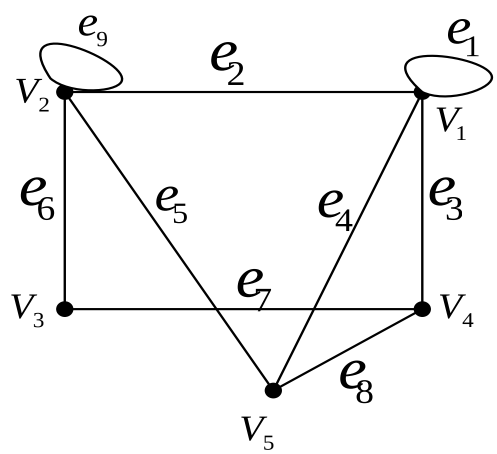
جــامعة بنـــــها
كلـية الـــــعلوم
قسم الرياضيات

نموذج اجابة امتحان
رياضيات متقطعة (225 ر)
التاريخ: 2019/1/13

السؤال الثالث (25 درجة) :-

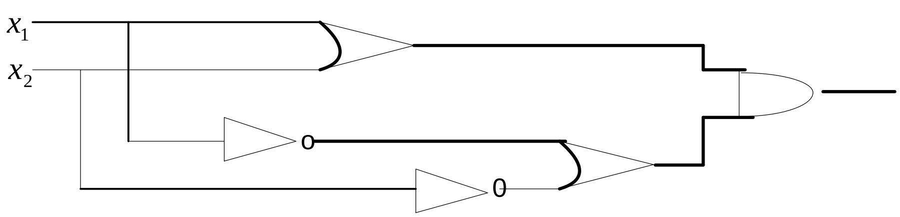
1- Let $f : A \to B$ and $g : B \to C$ be two functions **prove that**: **if** f and g are both injection **then** so, too is $g \circ f$

2- **Find** the matrix A^2, where A be the adjacency matrix, for the following graph: and **write** all edge sequences of length 2 joining v_2, v_2.



3- A relation $\equiv_5$ on the set Z is defined by $a \equiv_5 b$ if and only if $a - b = 5k$ for some $k \in Z$, show that $\equiv_5$ is an equivalent relation and describe the equivalence class [-5].

4- Determine the Boolean expression for the output of the following system of gates.



Good Luck ! انتهت أسئلة

مع أطيب تمنياتى بالتوفيق و النجاح

د. محمد السيد عبدالعال

جـــامعة بنـــــها
كلـية الـــــعلوم
قسم الرياضيات

نموذج اجابة امتحان
رياضيات متقطعة (225 ر)
التاريخ: 2019/1/13

نموذج اجابه لأمتحان رياضيات متقطعة (225 ر) لطلاب المستوى الثانى

(الدرجة الكلية 80 درجة)

اجابة السؤال الأول (35 درجة) :-

1- **Show** the following function is a **bijection** and find its **inverse**:

$$f : R-\{5\} \to R-\{2\},\ f(x)=\frac{2x+1}{x-5}\ \forall\, x\in R-\{5\}.$$

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To show that *f* is an injection we prove that, for all real numbers *x* and *y*, *f (x)* = *f (y)* implies *x* = *y*. Now *f (x)* = *f (y)*

$\Rightarrow \frac{2x+1}{x-5}=\frac{2y+1}{y-5}$ ....$\Rightarrow$ $x = y$ so *f* is injective.

To show that *f* is a surjection, let *y* be any element of the codomain *f* . We need to find x ∈ R-{5} such that f (x) = y. Let $x=\frac{1+5y}{y-2}$. Then *x* ∈ R-{5} and

$$f(x)=[2\frac{1+5y}{y-2}+1]\div[\frac{1+5y}{y-2}-5]=\frac{2+10y+y-2}{y-2}\div\frac{1+5y-5y+10}{y-2}=\frac{11y}{11}=y$$

so *f* is surjective.

To find f^{-1} we simply use its definition: if $y = f(x)$ then $x = f^{-1}(y)$.

Now

y = f (x) $\Rightarrow$ y = $\frac{2x+1}{x-5}$ $\Rightarrow$

$\Rightarrow x=\frac{1+5y}{y-2}$

$x = f^{-1}(y) = \frac{1+5y}{y-2}$.

Therefore the inverse function is f^{-1} : R-{2} →R-{5}, $f^{-1}(y) = \frac{1+5y}{y-2}$.

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2- Let $A=\{1,2,3,4\}$ and $R=\{(a,b):b\le a^2\}$ be a relation on *A*. **List** the elements of *R*, and **write** down its binary matrix. **Determine** which of the properties, reflexive, symmetric, transitive the relation *R* is satisfied.

الحـــــــــــــــل

$$\boldsymbol{R} = \{(\mathbf{1},\mathbf{1}),(\mathbf{2},\mathbf{1}),(\mathbf{2},\mathbf{2}),(\mathbf{2},\mathbf{3}),(\mathbf{2},\mathbf{4}),(\mathbf{3},\mathbf{1}),(\mathbf{3},\mathbf{2}),$$
$$(\mathbf{3},\mathbf{3}),(\mathbf{3},\mathbf{4}),(\mathbf{4},\mathbf{1}),(\mathbf{4},\mathbf{2}),(\mathbf{4},\mathbf{3}),(\mathbf{4},\mathbf{4})\}$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

R is reflexive not symmetric, transitive.

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3- For all propositions p, q, r, **Prove that:**

I. $(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv \overline{p}$ ***by laws***

II. $[(p \vee q) \rightarrow \overline{r}\,] \underline{\vee} (\overline{p} \vee \overline{q})$.

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$$(\overline{p} \wedge q) \vee (\overline{p \vee q}) \equiv (\overline{p} \wedge q) \vee (\overline{p} \wedge \overline{q}) \equiv \overline{p} \vee (q \wedge \overline{q}) \equiv \overline{p}$$

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$$[(p \vee q) \rightarrow \overline{r}\,] \underline{\vee} (\overline{p} \vee \overline{q})$$

p	q	r	$\overline{q}$	$\overline{r}$	$\overline{p}$	$(p \vee q)$	$(p \vee q) \rightarrow \overline{r}$	$\overline{p} \vee \overline{q}$	$(p \vee q) \rightarrow \overline{r}]\ \underline{\vee}\ (\overline{p} \vee \overline{q})$
1	1	1	0	0	0	1	0	0	0
1	1	0	0	1	0	1	1	0	1
1	0	1	1	0	0	1	0	1	1
1	0	0	1	1	0	1	1	1	0
0	1	1	0	0	1	1	0	1	1
0	1	0	0	1	1	1	1	1	0
0	0	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	1	1	0

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1- draw the diagram of a system of switches for which it is the switching function:

$f(x_1, x_2, x_3) = x_1[x_2(x_1 \oplus x_3) \oplus x_3 x_2]$

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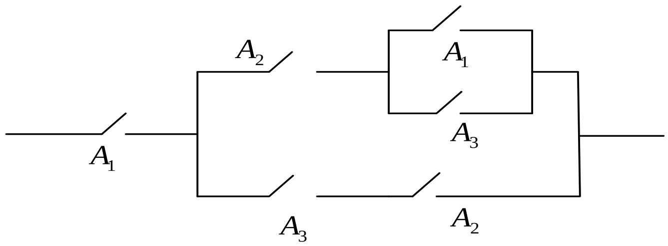

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السؤال الثانى (30 درجة) :-

1- Let $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Determine whether each of the following is true or false and give a brief justification.
$(i)\ B \in P(A)$, $(ii)\ B \in A$, $(iii)\ A \subseteq P(A)$, , $(iv)\ \emptyset \in P(A)$.

الحـــــــــــل

$(i)\ B \in P(A)$, True: B is a subset of A so B is an element of its power set.

$(ii)\ B \in A$, False: B is a set but the elements of A are numbers, so B is not an element of A.

$(iii)\ A \subseteq P(A)$, False: the elements of A are numbers whereas the elements of P(A) are sets (namely subsets of A). Hence the elements of A cannot also be elements of P(A), so A _⊆ P(A).

$(iv)\ \emptyset \subseteq P(A)$. True: since $\emptyset \subseteq A$. we have $\emptyset \in P(A)$.

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2- **Let** $(B, \oplus, *, \bar{}\ , 0, 1)$ be a Boolean algebra and for all $b_1, b_2 \in B$, **prove that**:
There is only one element $\overline{b_1} \in B$ such that $b_1 \oplus \overline{b_1} = 1$ and $b_1 * \overline{b_1} = 0$.

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Suppose that .b1 and .b2 are both complements of an element b of a Boolean algebra $(P(S), \cup, \cap, -, \varphi., S) = (B, \oplus, *, -, 0, 1)$. This means that

$$b \oplus .b_1 = .b_1 \oplus b = 1 , \qquad b \oplus .b_2 = .b_2 \oplus b = 1$$
$$b * .b_1 = .b_1 * b = 0 , \qquad b * .b_2 = .b_2 * b = 0., \qquad .b_1 = \overline{b_1}$$

Thus we have
.b1 = .b1 ∗ 1 (axiom B1)
= .b1 ∗ (b ⨁ .b2)

= (.b1 ∗ b) ⊕ (.b1 ∗ .b2) (axiom B4)
= 0 ⊕ (.b1 ∗ .b2)

= 0 ⊕ (.b2 ∗ .b1) (axiom B3)
= (.b2 ∗ b) ⊕ (.b2 ∗ .b1)
= .b2 ∗ (b ⊕ .b1) (axiom B4)
= .b2 ∗ 1
= .b2 (axiom B1).

We have shown that .b1 = .b2 and so we can conclude that the complement is unique.

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3- Let A , B , C **are sets, prove that:**

I. $A\times(B\cap C)=(A\times B)\cap(A\times C)$

II. $A-(B\cap C)=(A-B)\cup(A-C)$ ***by definitions***

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I. $A\times(B\cap C)=(A\times B)\cap(A\times C)$

Let (a, x) ∈ A × (B ∩ C). By the definition of the Cartesian product, this means that a ∈ A and x ∈(B ∩ C). Thus x ∈ X, so (a, x) belongs to A× B; and x ∈ C , so (a, x) belongs to A×C as well. Therefore (a, x) ∈ (A× B)∩(A×C), which proves that A × (B ∩ C) ⊆ (A × B) ∩ (A × C).

To prove the subset relation the other way round as well, let (a, x) ∈ (A × B) ∩ (A × C).

Then (a, x) ∈ (A × B), so a ∈ A and x ∈ B; and (a, x) ∈ (A × C), so a ∈ A and x ∈ C . Therefore a ∈ A and x ∈ (B ∩ C) which means that the ordered pair (a, x) belongs to the Cartesian product A × (B ∩ C). Hence (A × B) ∩ (A × C) ⊆ A × (B ∩ C).

The conclusion that the sets A × (B ∩ C) and (A × B) ∩ (A × C) are equal now

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II. $A-(B\cap C)=(A-B)\cup(A-C)$

First we show $A - (B \cap C) \subseteq (A - B) \cup (A - C)$.

Let $x \in A-(B\cap C)$. Then $x \in A$ and $x \notin B\cap C$. Hence $x \in A$ and either $x \notin B$ or $x \notin C$ (or both). Therefore either $x \in A$ and $x \notin B$ or $x \in A$ and $x \notin C$ (or both). It follows that $x \in A - B$ or $x \in A - C$ (or both). Hence $x \in (A - B) \cup (A - C)$. We have shown that if $x \in A - (B \cap C)$ then $x \in (A-B)\cup(A-C)$. Therefore $A-(B\cap C) \subseteq (A-B)\cup(A-C)$.
Secondly we must show that $(A - B) \cup (A - C) \subseteq A - (B \cap C)$.

Let $x \in (A - B) \cup (A - C)$. Then $x \in A - B$ or $x \in A - C$ (or both) so $x \in A$ and $x \notin B$ or $x \in A$ and $x \notin C$ (or both). Hence $x \in A$ and either $x \notin B$ or $x \notin C$ (or both) which implies $x \in A$ and $x \notin B \cap C$. Therefore $x \in A - (B \cap C)$. We have shown that if $x \in (A - B) \cup (A - C)$ then $x \in A - (B \cap C)$. Therefore $(A - B) \cup (A - C) \subseteq A - (B \cap C)$.
Finally, since we have shown that each set is a subset of the other, we may conclude $(A - B) \cup (A - C) = A - (B \cap C)$.

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4- **Defined** *the complete graph* K_n , *the complete bipartite graph* $K_{r,s}$ and *Eulerian path*, and for **which values** of n, r, s , the graphs $K_n , K_{r,s}$ are **Eulerian**?

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A complete graph is a simple graph in which every pair of distinct vertices is joined by an edge.
A complete bipartite graph is a bipartite graph such that every vertex of V_1 is joined to every vertex of V_2 by a unique edge.
An Eulerian path in a graph G is a closed path which includes every edge of G . A graph is said to be Eulerian if it has at least one Eulerian path.

The complete graph K_n is $(n-1)$-regular—every vertex has degree $n-1$. Since it is connected, K_n is Eulerian if and only if n is odd (so that $n - 1$ is even).

A complete bipartite graph $K_{r,s}$ is Eulerian if and only if r,s is even.

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السؤال الثالث (25 درجة) -:

1- Let $f : A \to B$ and $g : B \to C$ be two functions **prove that**: **if** *f* and *g* are both injection **then** so, too is $g \circ f$

suppose *f* and *g* are injections. Let $a, a_1 \in A$, $b = f(a)$ and $b_1 = f(a_1)$.
Then $g \circ f(a) = g \circ f(a_1) \Rightarrow g(f(a)) = g(f(a_1))$
$g(b) = g(b_1)$
$b = b_1$ (since g is injective)

$\Rightarrow f(a) = f(a_1)$ (since $f(a) = b$, $f(a_1) = b_1 \Rightarrow$)

$\Rightarrow a = a_1 \Rightarrow$ (since f is injective).

Hence $g \circ f$ is an injection.

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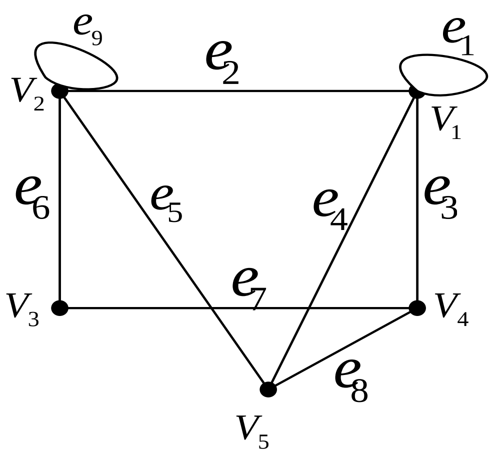
2- **Find** the matrix A^2, where A be the adjacency matrix, for the following graph:

and **write** all edge sequences of length 2 joining v_2, v_2.



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$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 4 & 3 & 2 & 2 & 3 \\ 3 & 4 & 1 & 3 & 2 \\ 2 & 1 & 2 & 0 & 2 \\ 2 & 3 & 0 & 3 & 1 \\ 3 & 2 & 2 & 1 & 3 \end{pmatrix}$$

$e_9\,e_9$; $e_2\,e_2$; $e_5\,e_5$; $e_6\,e_6$.

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3- A relation $\equiv_5$ on the set Z is defined by $a \equiv_5 b$ if and only if $a - b = 5k$ for some $k \in Z$, show that $\equiv_5$ is an equivalent relation and describe the equivalence class [-5].

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In this case $a \equiv_5 b$ if and only if a $-$b = 5k for some integer k; that is, if and only if there exists an integer k such that a = 5k + b.

Firstly, R is reflexive since a-a = 5 0, Secondly, if $a \equiv_5 b$ i.e. a-b =5k then b-a = − 5k so implies $b \equiv_5 a$ therefore $\equiv_5$ is symmetric.

Thirdly, suppose $a \equiv_5 b$ and $b \equiv_5 c$; then there exist integers k such that a-b =5k and b-c =$5k_1$.

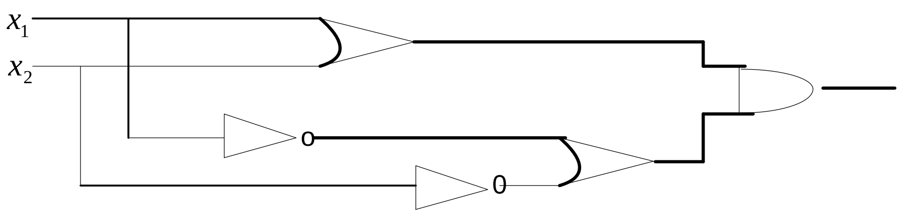
Combining these two equations gives a-c =$5(k-k_1)$ therefore $a \equiv_5 c$

where (k-k1)is an integer. Thus $a \equiv_5 b$ and $b \equiv_5 c$ implies $a \equiv_5 c$ so $\equiv_5$ is transitive.

Therefore [p] = {q ∈ z : q = 5k + p, for some k ∈ z}.

[-5] = {q ∈ z : q = 5k + -5, for some k ∈ z}.

4- Determine the Boolean expression for the output of the following system of gates.



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$$(x_1 \oplus x_2)(\overline{x_1} \oplus \overline{x_2}).$$